

# Curve Sketching

4.6  
67

$$f(x) = 3x^4 - 4x^3 - 12x^2 - 1$$

$$f'(x) = 12x^3 - 12x^2 - 24x$$

$$= 12x(x^2 - x - 2)$$

$$= 12x(x-2)(x+1)$$

Critical points where  $f'(x)$  undefined — none

① VERT. TAN LINE IF  
 $f'(x) = \text{undef.}$   
AND  $f(x) = \text{def.}$

② NO TAN LINE IF  $f'(x)$  also undef.

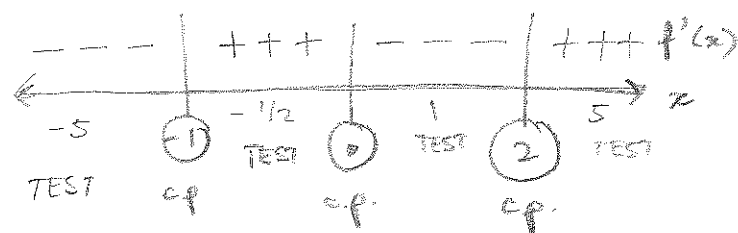
—————  $f'(x) = 0$  where  $x = 0, x = 2, x = -1$ .

→ HORIZONTAL TANGENT LINES

$$f(0) = -1 \quad (0, -1) \text{ LOCAL MAXIMUM}$$

$$f(-1) = 3 + 4 - 12 - 1 = -6 \quad (-1, -6) \text{ LOCAL MINIMUM}$$

$$f(2) = 3(16) - 4(8) - 12(4) - 1 = -33 \quad (2, -33) \text{ LOCAL MINIMUM}$$



$f(x)$  increasing on  $(-1, 0) \cup (2, +\infty)$

$f(x)$  decreasing on  $(-\infty, -1) \cup (0, 2)$

so  $(0, -1)$  — local max, horiz. tan line

$(-1, -6)$  — local min, —————

$(2, -33)$  — local min, —————

$$f''(x) = 36x^2 - 24x - 24$$

$$= 12(3x^2 - 2x - 2)$$

$$\frac{2 \pm \sqrt{4 - 4(-2)(3)}}{2(3)} = \frac{2 \pm 2\sqrt{7}}{6}$$

P.P.I. occurring when  $f''(x)$  is undefined — none

————  $f''(x) = 0$  occur when  $3x^2 - 2x - 2 = 0$ , i.e.,

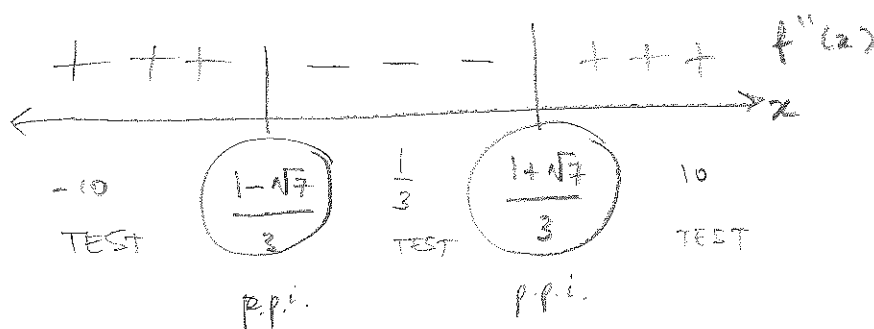
when  $x = \frac{2 \pm \sqrt{(-2)^2 - 4(-2)(3)}}{2(3)}$

$$= \frac{1}{3} \pm \frac{1}{3} \sqrt{1 - (-2)(3)}$$

$$= \frac{1 \pm \sqrt{7}}{3}$$

$$f\left(\frac{1+\sqrt{7}}{3}\right) \approx -19.4$$

$$f\left(\frac{1-\sqrt{7}}{3}\right) \approx -3.7$$



$f$  is concave down on  $\left(\frac{1-\sqrt{7}}{3}, \frac{1+\sqrt{7}}{3}\right)$

———— up on  $(-\infty, \frac{1-\sqrt{7}}{3}) \cup \left(\frac{1+\sqrt{7}}{3}, +\infty\right)$

