LECTURE 12: FINAL EXAM REVIEW.

- HW 6 due Weds. 11:59 p.m.
- [redacted] done grading HW 5

- Final exam:
  - 90 total points + 10 bonus points
  - 9 questions (may be multi-part) + 1 bonus question (may also be multi-part)
  - no calculators / cell phones / etc.
  - no "cheat sheets", no notes, no books
  - not cumulative (except in the sense that all mathematics is cumulative) — only on 11.1 – 11.5, 12.1 – 12.5, 13.1 – 13.4

TEXT SECTIONS — PROBLEM CATEGORIES

11.1 : Parametrizations of plane curves

- Parametric eq'ns
- ~~Cycloids~~
- ~~Brachistochrones & Tautochrones~~

[P]
- Finding Cartesian from Parametric
- Finding parametric eq'ns
- ~~Distance using parametric eq'ns~~

11.2 : Calculus w/ parametric curves

[F] · Tangents and areas

[F] · Length of a parametrically def. curve
- ~~Length of a curve $y=f(x)$~~
- ~~Arc length differential~~

[F] · Areas of surfaces of revolution

[P]
- Tangents to parametrized curves
- ~~Implicitly defined parametrizations~~
- Area
- Lengths of curves
- Surface area
- ~~Centroids~~

11.3 : Polar coordinates

- Def'n of polar coordinates
- Polar eq'ns & graphs

[F] · Relating polar & Cartesian coords

- Polar coordinates

[P]
- Polar to Cartesian coords
- Cartesian to polar coords

Same "P"

[P]
- Graphing sets of polar coord. points
- Polar to cartesian eq'ns
- Cartesian to polar eq'ns

11.4 : Graphing polar coord. eq'ns

- Symmetry
- Slope
- ~~Converting graph from (r,θ) to (x,y) plane~~

[P]
- Symmetries & polar graphs
- Slopes of polar curves in xy-plane
- Graphing Limaçons
- ~~—~~ polar regions & curves in the xy-plane

TEXT SECTIONS — PROBLEM CATEGORIES

11.5: Areas & Lengths in polar coordinates

F
- Area in the plane
- Same formulas from 11.2
- Length of a polar curve

P
- Finding polar areas
- Finding lengths of polar curves

→ "QUESTIONS TO GUIDE YOUR REVIEW" №s 1–13, p. 699

12.1: 3D coordinate system

F
- Distance & spheres in space

P
- Geometric interpretat'ns of eq'ns
- ——— & ——— inequalities
- Inequalities to describe sets of points
- ~~Distance btwn. points~~
- Spheres

12.2: Vectors

- Component form
- Vector algebra operat'ns (add'n, scalar mult.)

F
- Unit vectors
- ~~Midpoint of a line segment~~
- Applications – ONLY EX. 9 p. 715

- Vectors in the plane (2D) (finding comp. form & magnitude)
- Vectors in space (3D)

P
- Geometric representations (head-to-tail add'n)

P F
- Length & direc'n
- ~~Direc'n & midpoints~~

12.3: The dot product

F
- Angle btwn. vectors.

F
- Orthogonal vectors

(Dot product properties)

F F
- Vector projec'n

F
- Work

(✱) Dot product & projec'ns
(Angle btwn. vectors)
- ~~Eq'ns for lines in the plane~~
- ~~Angles btwn. lines in the plane~~
- Work

TEXT SECTIONS — PROBLEM CATEGORIES

12.4: The cross product

- [P] Cross prod. of vectors in space
  - ~~$|\vec{u}\times\vec{v}|$ is the area of parallelogram~~
- [F] determinant formula for $\vec{u}\times\vec{v}$.
- [F] Torque
- ~~Triple scalar or box product~~

- Cross product calculation
- ~~Triangles in space~~ [P]
- ~~Triple scalar prods~~
- ~~Area of a parallelogram~~
- ~~Area of a triangle~~

12.5: Lines and planes in space

- [F] Lines & line segments in space
  - vector & parametric line eq'ns
- [F] Distance from a pt. to a line in space
- [F] An eq'n for a plane in space
- [F] Lines of intersec'n
- [F] Distance from a pt. to a plane
- [F] Angles btwn. planes

- Lines & line segments [P]
- Planes [P]
- Distances [P]
- Angles [P]
- Intersecting lines & planes [P]

[P]

→ "QUESTIONS TO GUIDE YOUR REVIEW" Nº 1-14, p. 745

13.1: Curves in space & their tangents

- Particle paths
- ~~Limits & continuity~~
- ~~Derivatives & motion~~
- (Differentiat'n rules)
- ~~Vector fns. of constant length~~

- Motion in the plane
- Motion in space
- Tangents to curves [P]
- Theory & examples
  - Motion along circle, (parabola, ~~ellipse~~)

13.2: Integrals of vector fns.; projectile motion

- Integrals of vec. fns.
- The vec. & parametric eq'ns for IDEAL proj. motion [F] [P]
- ~~Proj. motion w/ wind gusts~~

- Integrating vec. fns.
- Initial value probs.
- ~~Motion along straight line~~
- Projectile motion (ideal; ~~w/ linear drag~~)

TEXT SECTIONS — PROBLEM CATEGORIES

13.3: Arc length in space

- F: Arc length along a space curve
- Speed on a smooth curve
- F: Unit tangent vector
- Finding tan. vecs & lengths
- Arc length parameter
- Theory & examples (arc length; helices; ~~ellipses~~) P

13.4: Curvature & normal vectors of a curve

- Curvature of a plane curve F
- Principal unit normal vector F
- ~~Circle of curvature for plane curves~~

(Curvature & normals for space curves)

- Plane curves } same formulas
- Space curves }
- More on curvature

→ "QUESTIONS TO GUIDE YOUR REVIEW", Nº 1–10, p. 788

FORMULAS TO KNOW

11.2:
- Derivative of parametric curve: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$
- Length ($y=y(t)$, $x=x(t)$): $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
- Area of surface of revolution (x-axis / y-axis): $A = \int_a^b 2\pi \begin{Bmatrix} x \\ y \end{Bmatrix} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

11.3:
- Polar coordinate formulas: $x = r\cos(\theta)$, $y = r\sin(\theta)$

12.2:
- Vector magnitude & direction: $\vec{v} = \langle v_1, v_2, v_3 \rangle = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$

$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ ← SCALAR !!

$\hat{v} = \vec{v}/|\vec{v}|$

12.3:
- Dot product $\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$
- Work $W = \vec{F}\cdot\vec{D}$
- Vector projection projecting $\vec{u}$ onto $\vec{v}$: $\text{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v}$ P
- Angle btwn. two vectors $\theta = \arccos\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)$
- Orthogonality $\vec{u}\cdot\vec{v} = 0 \Leftrightarrow \vec{u}$ and $\vec{v}$ are orthogonal.

FORMULAS TO KNOW, CT'D.

12.4: • Cross product $\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}\hat{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}\hat{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}\hat{k}$

$= (u_2v_3 - u_3v_2)\hat{i} - (\quad)\hat{j} + (\quad)\hat{k}$

• Torque $\vec{T} = \vec{F}\times\vec{D}$

• Parallel vectors $\vec{u}\times\vec{v} = \vec{0} \iff \vec{u} \parallel \vec{v}$ ("$\parallel$" is "parallel to")

12.5: • Eq'n for a line in space

$\vec{r}_0$ = a pt. on the line, $\vec{v} \parallel$ the line

$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$

• Distance from pt. to line

P on the line, $\vec{v} \parallel$ the line, S is a pt. in space

$$d = \frac{|\vec{PS}\times\vec{v}|}{|\vec{v}|}$$

• Eq'n for a plane in space

$\langle A,B,C\rangle = \vec{n}$ is normal to plane, P_0 ((x_0, y_0, z_0)) is a pt. on plane

$$\vec{n}\cdot\vec{P_0P} = 0$$

$$A(x-x_0)+B(y-y_0)+C(z-z_0) = 0$$

• Dist. from a pt. to a plane

$\vec{n}$ is normal to plane; P is on plane; S in space:

$$d = \left|\vec{PS}\cdot\frac{\vec{n}}{|\vec{n}|}\right|$$

• Lines of intersec'n

$\vec{n}_1$ is normal to Plane 1, $\vec{n}_2$ norm. pln. 2; a vector $\parallel$ to the line of intersec'n is $\vec{n}_1\times\vec{n}_2$

13.1 • Ideal projectile motion eq'n (DON'T MEMORIZE)

$\vec{r}(t) = \langle (\underbrace{v_0}_{\text{init. speed}}\cos\alpha)t, (v_0\sin\alpha)t - \frac{1}{2}gt^2\rangle$ (α = $\measuredangle$ of firing)

13.3 • Arc length along a space curve

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

• Unit tangent vector

$$\hat{T} = \frac{\vec{v}}{|\vec{v}|}$$

13.4 • Curvature

$$\kappa = \frac{1}{|\vec{v}|}\frac{d\hat{T}}{dt}$$

• Principal unit normal vector

$$\hat{N} = \frac{d\hat{T}/dt}{\left|d\hat{T}/dt\right|}$$

TECHNIQUES TO KNOW.

11.1: • Graphing parametric eq'ns
EX. 1, 3, 7
• Deparametrizing curves
EX. 2, 4, 5, 6
• Parametrizing curves
Probs. 19-38

11.2: • Area EX. 3

12.2: • Adding & scalar mult. of vectors
EX. 3
• Head-to-tail add'n
EX. 3
• Effective force
EX. 2

12.3: • "Geometric" ~~cross~~ dot product
EX. 1, 2, 3

12.4: • "Geometric" cross product
EX. 1, 2, 4 – esp. diagram on 1st pg. of sec'n.

12.5: • Visualizing planes + lines in space
EX. 1 – 12

13.1: • Tangents to curves PROBS. 19-22
• Motion along figures PROBS. 23-25

13.2: • Initial value problems EX. 3; PROBS 11-16
• Ideal & ~~non-ideal~~ projectile motion
EX. 4, ~~5~~; PROBS 19-36

13.3: • Geometric interpretation of $\hat{T}$
EX. 3

13.4. • _____ $\hat{N}$
EX. 3, 6
• _____ K
EX. 1, 2, 5

11.4 • Graphing polar
EX. 1, 2, 3

11.3 • "Polarizing" curves
EX. 2, 4, 5
• "Depolarizing" curves
EX. 4, 6

11.5 • Area + length
EX. 1, 2, 3