Lecture 5 : Example Problems.

10.7

Intervals of convergence.

(Gen. form : $\sum_{n=0}^{\infty} c_n (x-a)^n$)

$\neq$

3) $\sum_{n=0}^{\infty} (x+5)^n$ is a geometric series $\sum_{n=0}^{\infty} a_g \cdot r^n$, with $a_g = 1$ and $r = x+5$, so it converges when $|r| = |x+5| < 1$, i.e., $x \in (-4, 6)$, and $\sum_{n=0}^{\infty} (x+5)^n = \frac{1}{1-(x+5)} = \frac{-1}{x+4}$.

$a=0$, $c_n = 2^n$

6) $\sum_{n=0}^{\infty} (2x)^n$ is a geom. series $\sum_{n=0}^{\infty} a_g \cdot r^n$, with $a_g = 1$ and $r = 2x$, so it converges when $|r| = |2x| < 1$, i.e., when $x \in (-1/2, 1/2)$, and it converges to $1/1-r = 1/1-2x$ there.

Using geom. series

50) Use geom. series to represent $f(x) = \frac{5}{3-x}$ ~~and~~ $\cancel{g(x) = \frac{3}{x-2}}$ as power series about $x = 2$, and find intervals of convergence.

(a) $f(x) = \frac{5}{3-x}$. Let's look for r s.t. $1-r = 3-x$. Then $r = x-2$ works, so $f(x) = \frac{5}{1-(x-2)}$, which is the sum of the geometric series $\sum_{n=0}^{\infty} a r^n$, with $a = 5$, $r = x-2$, i.e., it is the sum of $\sum_{n=0}^{\infty} 5(x-2)^n$, when this series converges. The series converges when $|r| = |x-2| < 1$, i.e., when $x \in (-1, 3)$.

10.8

Find Taylor poly's of order 0, 1, 2, 3 :

2) $f(x) = \sin(x)$, $a = 0$

$$P_0(x) = f^{(0)}(0) = \sin(0) = 0$$

$$\begin{cases} P_1(x) = 0 + \frac{f^{(1)}(0)}{1!}(x-0)^1 = 0 + f^{(1)}(0)\, x = 0 + \cos(0)\cdot x = x \\ P_2(x) = 0 + x + \frac{f^{(2)}(0)}{2!}x^2 = 0 + x + (-\sin(0))x^2 = x \end{cases}$$

$$\begin{cases} P_3(x) = 0 + x + 0 + \frac{f^{(3)}(0)}{3!}x^3 = x + \frac{(-\cos(0))}{3!}x^3 = x - \frac{x^3}{3!} \\ P_4(x) = x - \frac{x^3}{3!} \end{cases}$$

$$\begin{cases} P_5 = P_6 \end{cases}$$

L5 probs, ct'd.

10.8

27) $f(x) = \frac{1}{x^2}$, $a = 1$

$$f^{(0)}(x) = \frac{1}{x^2} \Rightarrow f^{(0)}(1) = \frac{1}{1^2} = 1$$

$$f'(x) = \frac{-2}{x^3} \Rightarrow f'(1) = \frac{-2}{1^3} = -2$$

$$f''(x) = \frac{3\cdot 2}{x^4} \Rightarrow f''(1) = \frac{3\cdot 2}{(1)^4} = 3\cdot 2$$

$$f'''(x) = \frac{-4\cdot 3\cdot 2}{x^5} \Rightarrow f'''(1) = -4\cdot 3\cdot 2$$

$$\vdots \qquad\qquad \ddots$$

$$f^{(n)}(x) = \frac{(-1)^n (n+1)!}{x^{(n+2)}} \qquad f^{(n)}(1) = (-1)^n (n+1)!$$

So, the Taylor series generated by $f(x) = \frac{1}{x^2}$ about $a = 1$ is:

$$\sum_{n=0}^{\infty} \underbrace{\frac{f^{(n)}(a)}{n!}}_{c_n} (x-a)^n = \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)!}{n!} (x-1)^n$$

$$= \sum_{n=0}^{\infty} \underbrace{(-1)^n (n+1)}_{c_n} (x-1)^n$$