Monday, March 20, 2017 (L22).

Housekeeping.

- Week 5 reading {summary, discussion post} due 11:59 p.m. tonight
- Homework (A13) due Wednesday in class
- Quiz 6 on Wednesday
- End of withdraw period on Wednesday
- Extra credit (Wage Gap Rebuttal) due 11:59 p.m. Monday (Mar. 27)
- Exam 2 is one week from Friday (it will be March 31)
  - Review session on Wednesday March 29.
- Book project: essay prompt posted under "Announcement" on book group page on Canvas.

Last time.

- Rules of probability for compound events
- Event complements
- Mutually exclusive events
- Conditional probability

Questions?

This time: More conditional probability.

<u>Recall</u>: $P(A|B)$ is read "the probability of A, given B",

and it represents the probability that A will happen, <u>given that</u> B has already happened (or will have already happened).

or...

the probability that A will happen <u>if</u> B happens

or...

the probability that A will happen, <u>conditioned upon</u> B happening.

There are several ways of expressing conditional statements in English, and you will, doubtless, come across more.

<u>Some examples</u>...

The following table shows the # of athletes who stretch before exercising, and how many had injuries in the past year:

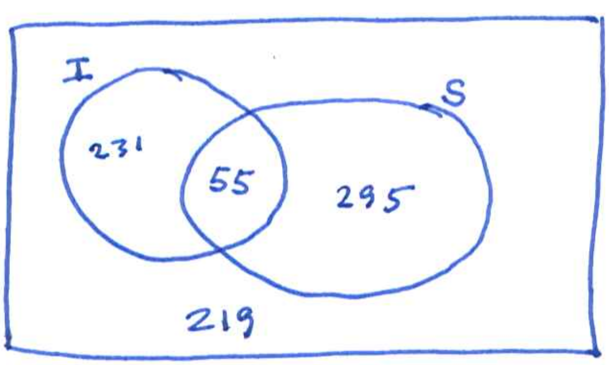
	INJURY	NO INJURY	TOTAL
STRETCHES	55	295	350
DOESN'T STRETCH	231	219	450
TOTAL	286	514	900

(p. 192, OpenStax)
(Try It 3.20)

Question: what goes here ?

Let I be the event that an athlete has had an injury in the past year, and
let S be the event that an athlete stretches before practice.

The Venn diagram:



Note: the table itself is also a fine Venn diagram:

	Injured	not I	total
S	55	295	350
not S	231	219	450
tot.	286	514	800

Questions: • What is $P(I)$?

$$P(I) = \frac{\# I}{\text{total } \#} = \frac{286}{800}$$

• What is $P(I \mid S)$?

$$P(I \mid S) = \frac{\# I \text{ and } S}{\# S} = \frac{55}{350}$$

• What is $P(S) = \frac{350}{800}$.

• What is $P(I \text{ and } S) = \frac{55}{800}$.

Recall that $P(I) = \dfrac{286}{800}$, $P(s) = \dfrac{350}{800}$,

and $P(I \text{ and } s) = \dfrac{55}{800}$.

We calculated $P(I|s) = \dfrac{55}{350}$ directly from the table

(this shows how cond'l probability can be computed by "reducing the sample space").

But could we also say...

$$P(I|s) = \frac{55/800}{350/800} = \frac{P(I \text{ and } s)}{P(s)} \quad ✓$$

$$= \frac{55}{350}.$$

In general:

$$P(A|B) = \frac{\#\,A \text{ and } B}{\#\,B}$$

$$= \frac{\#\,A \text{ and } B}{\#\,B} \underbrace{\left(\frac{1/\#S}{1/\#S}\right)}_{1}$$

$$= \frac{\#\,A \text{ and } B/\#S}{\#\,B/\#S}$$

$$\boxed{P(A|B) = \frac{P(A \text{ and } B)}{P(B)}}$$

So, if you know $P(A \text{ and } B)$ and $P(B)$, then you can find $P(A|B)$ without even knowing the size of the sample space.

Questions?

	I	not I	tot.
S	55	295	350
not S	231	219	450
tot.	286	514	800

Questions...

$P(\text{not } S) = 450/800$

$P(\text{not } I) = 514/800$

$P(S \mid I) = 55/800$

$P(I \mid \text{not } S) = 231/450$

$P(S \mid \text{not } I) = 295/514$

$P(\text{not } S \mid I) = 231/286$

L22, ct'd.

~~[illegible]~~ what is...

Probability that an athlete who didn't stretch is injured?

$$P(I \mid \text{not } S) = 231/450$$

prob. tht. an athlete who stretched isn't injured?

$$P(\text{not } I \mid S) = 295/350$$

prob. tht. an athlete who is injured stretched?

$$P(S \mid I) = 55/286$$

prob. tht. an athlete who is injured didn't stretch?

$$P(\text{not } S \mid I) = 231/286$$

Independent events.

Two events are independent when one of them occurring has no effect on the probability of the other occurring.

That is... if A & B are independent, then B having occurred (or not) doesn't change/affect the probability tht. A occurs (or not), and vice-versa.

That is... if A & B are independent,

$$P(A|B) = P(A)$$

— and —

$$P(B|A) = P(B).$$

Recall, though, that regardless of whether A & B are independent,

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{and} \quad P(B|A) = \frac{P(B \text{ and } A)}{P(A)},$$

If A & B are indep., then

$$P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)$$

i.e.,
$$P(A) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{and} \quad P(B) = \frac{P(A \text{ and } B)}{P(A)}.$$

So, if A & B are independent,

what is $P(A \text{ and } B)$?

[if $z = \frac{x}{y}$ then $x = z \cdot y$]

So $\boxed{P(A \text{ and } B) = P(A)\,P(B) \quad \text{if } A \text{ \& } B \text{ are indep.}}$

Note: Independence + mutual exclusivity are different!!