L24: March 24, 2017.

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Housekeeping: • A14 was not posted on Canvas before Today — so will not be due until Monday in class

• Also due Monday: Week 6 reading (11:59 p.m., Canvas)

• Extra Credit: 11:59 p.m. Monday on Canvas

• Exam 2: Friday, March 31 in class

• Review on Wednesday

• Book essay prompts on Canvas.

Last time: Independent events

Questions?

This time: More on independent events

Multiplication rules

Group problem solving?

Recall: For independent events A & B,

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

Example. You're flipping two coins – a quarter and a nickel. How the quarter lands has no bearing on how the nickel lands (and vice versa) – so the two events are independent.

Therefore, $P(\text{quarter is } H \text{ and nickel is } H) =$

$S = \{(H,H) \leftarrow, (H,T), (T,H), (T,T)\}$

$$= P(\text{quarter is } H) \cdot P(\text{nickel is } H)$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Example. What is the probability of getting no six on a single throw of a die?

$$P(\text{not getting a six}) = \frac{5}{6}.$$

Example. What is the probability of getting no six on four rolls of a fair die?

$$P(\bar{o}\text{ six on 4 rolls}) = P(\bar{o}\text{ six }1^{st}\text{ roll}) \cdot P(\bar{o}\ 6\text{ on }2^{nd}) \cdot P(\bar{o}\ 6\text{ on }3^{rd}) \cdot P(\bar{o}\ 6\text{ on }4)$$
$$= \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = \boxed{\left(\frac{5}{6}\right)^4} \approx 0.482 = 48.2\%.$$

Example. What is the probability of getting a six (at least one six) in 4 rolls of a fair die?

$$P(\text{at least one 6 on 4 rolls}) = P\left((\bar{o}\text{ six on 4 rolls})^c\right) = 1 - \boxed{P(\bar{o}\ 6\text{ on 4 rolls})} = 1 - \left(\frac{5}{6}\right)^4 = \frac{6^4 - 5^4}{6^4} \approx 0.518$$

or, 51.8%.

Example. Let B_i be the event that no double six is thrown on the i^{th} roll of a pair of dice. $P(B_i) = \frac{35}{36}$ for all i, and each roll is independent from all the others.

$$\text{So } P(B_1 \text{ and } B_2 \text{ and } B_3 \text{ and} \dots \text{and } B_{24}) = P(B_1)\cdot P(B_2)\cdot P(B_3)\cdots P(B_{24}) = \underbrace{\left(\frac{35}{36}\right)\left(\frac{35}{36}\right)\cdots\left(\frac{35}{36}\right)}_{24\text{ times}}$$
$$= \left(\frac{35}{36}\right)^{24}.$$

i.e., $P(\text{not rolling a double six in 24 throws}) = \left(\frac{35}{36}\right)^{24}$.

So $P(\text{rolling at least one double six in 24 throws}) = 1 - \left(\frac{35}{36}\right)^{24}$

$$\approx 0.491 = 49.1\%.$$

So the probability of rolling at least one double six in 24 throws is 49.1%,

and

the probability of rolling at least one six in 4 throws of a single die is

51.8%.

This was the Chevalier de Méré's problem (see Cartoon Guide) !

the sol'n hinged on independent events, and rules for event complements.

Multiplication rules.

Recall: $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$

$$\boxed{P(A \text{ and } B) = P(A|B) \cdot P(B)}$$

"Multiplication rules"

By the same token, $P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)}$,

so $\boxed{P(A \text{ and } B) = P(B|A) \cdot P(A).}$

Example. A jury consists of 9 men and 3 women. If 2 are randomly selected, what's the prob. tht. they'll both be women?

Let A be the event tht. the 1st juror selected is a woman;

let B be the event — " — 2nd ——— " ——.

The question is asking us to find $P(A \text{ and } B)$.

We can find $P(A) = \dfrac{\#\text{ women on jury}}{\#\text{ of people on jury}}$

$$= \frac{3}{12} = \frac{1}{4}.$$

We can also find $P(B|A)$. If A occurs — i.e., if the 1st juror picked is a woman, then the remaining jury is 9 men and 2 women — so,

$$P(B|A) = \frac{\#\text{ women remaining}}{\#\text{ jurors remaining}} = \frac{2}{11}.$$

$P(A \text{ and } B) = P(B|A) \cdot P(A)$, by the mult. rule above

$$= \frac{2}{11} \cdot \frac{1}{4} = \frac{1}{11 \cdot 2} = \frac{1}{22} = 0.04545\ldots \approx \underline{\underline{4.5\%}}.$$