L36: Fri., Apr. 28

Housekeeping:
- A21 to be due ~~[illegible]~~ Monday.
- Infographic extra credit — Monday is last day
- Poisson extra credit – complete before exam
- Final essay due 11:59 p.m. May 5 on Canvas
- Final exam Fri, May 5 $\begin{cases} \text{10:30, Sec. 01} \\ \text{1:00, Sec. 02} \end{cases}$

Last time: Cts. r.v. & PDF ("probability density function") properties

This time: More on cts r.v. & the uniform distribution

Recall: Continuous r.v. MUST have their PDFs ~~BE~~ SHOWN ON A GRAPH, or given by a function — cannot list the values a cts. r.v. takes on, so cannot give probability distribution in a table.

~~[illegible]~~

NOTE: There is a fine distinction btwn. discrete & cts. r.v. – for example, the last question on A20...

If age (a cts. r.v.) is measured by the nearest year, then the number of years is discrete (because you can count to it — finite precision).

	discrete		cts.
Remember:	"count nouns"	vs.	"mass nouns"
	– fewer		– less
	– many		– much
	– few		– little

$\boxed{\text{"His age is less; he's lived fewer years."}}$

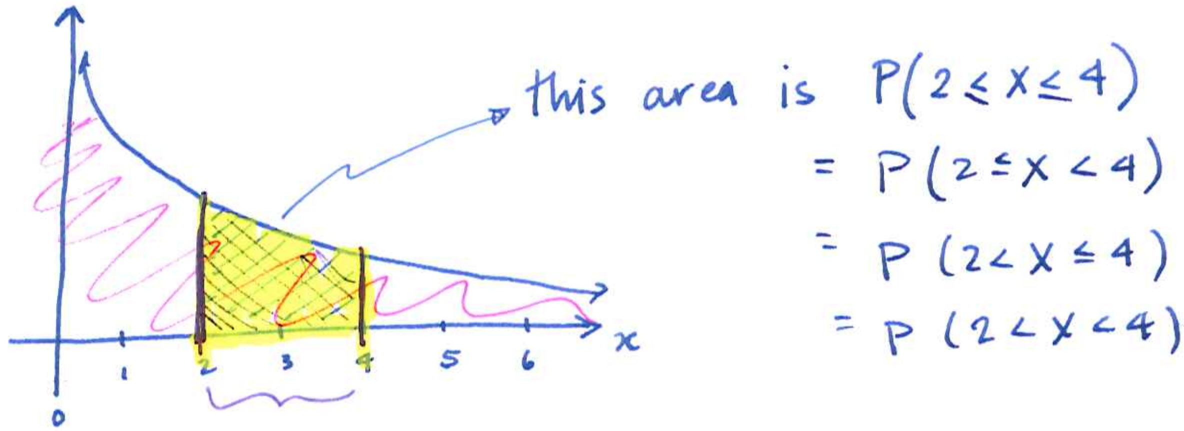
Recall: For a curve to be a PDF, it must satisfy:

1. Never drops below horizontal axis
2. Area below curve (& above axis) must be 1.

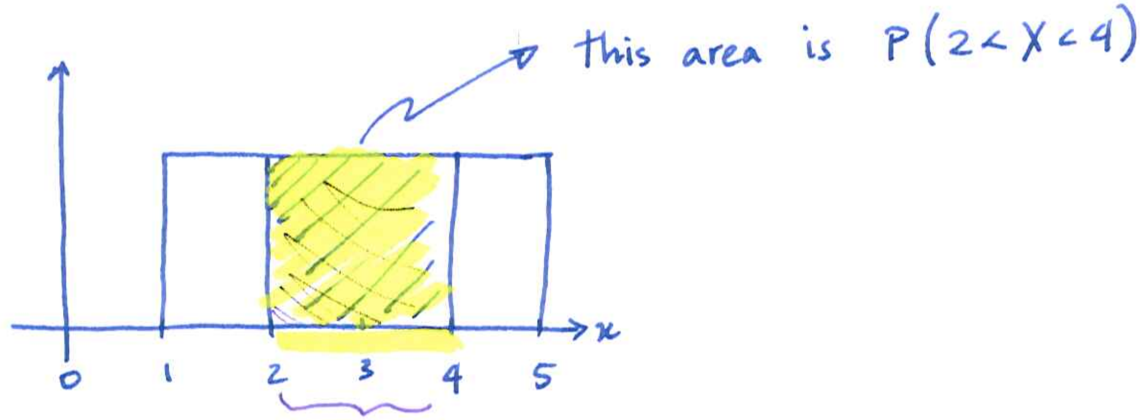
These requirements suggest a correspondence between AREA and PROBABILITY — and that is the case for cts. r.v. :

For cts. r.v., probability is found for INTERVALS of x-values (rather than for individual x-values) and is equal to AREA UNDER the PDF & OVER an interval.

For example:



or



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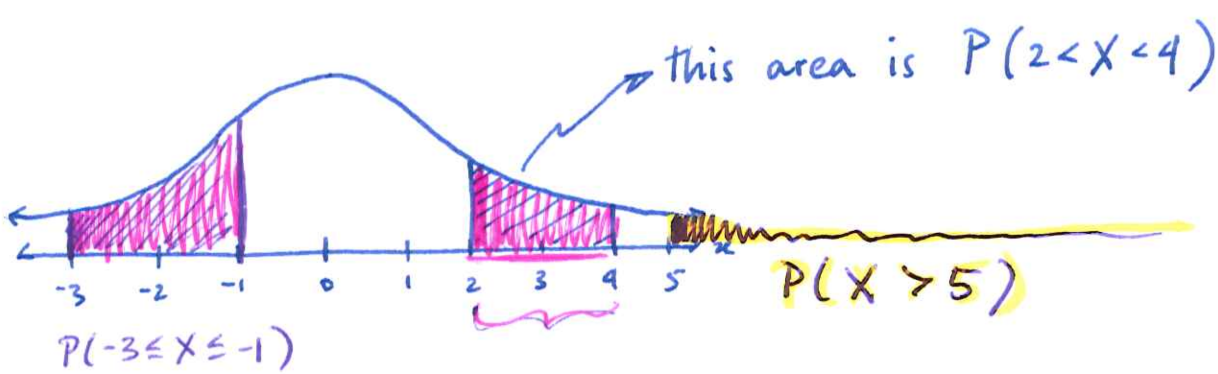


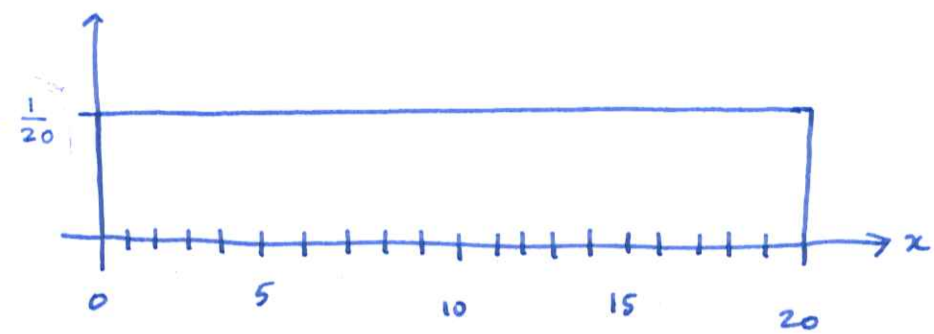
The above curves are all PDFs for different random variables X. The 1st graph shows an EXPONENTIAL distribution; 2nd shows a UNIFORM distrib'n; 3rd shows a NORMAL distribution.

EXAMPLE. Suppose X is uniformly distributed over the interval $[0, 20]$.

$$X \sim \text{unif}(0, 20)$$

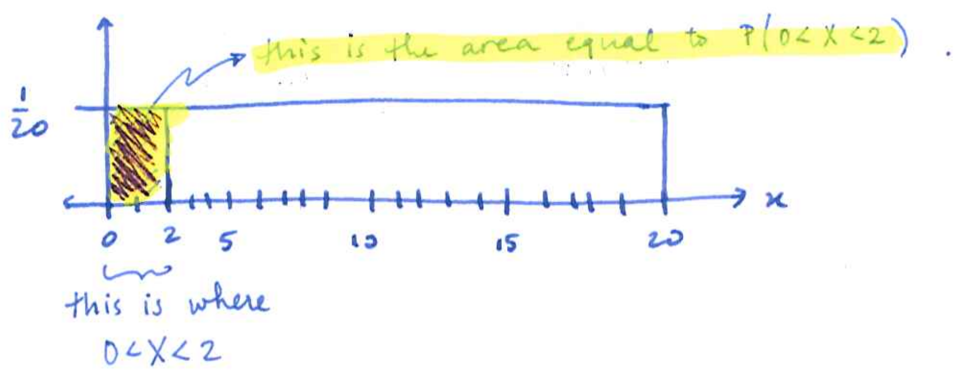
Then X has PDF:

$\frac{1}{20-0} = \frac{\text{area}}{\text{base}} = \frac{1}{20}$



Suppose we want to find the probability $P(0 < X < 2)$.

That's the same as finding the area:



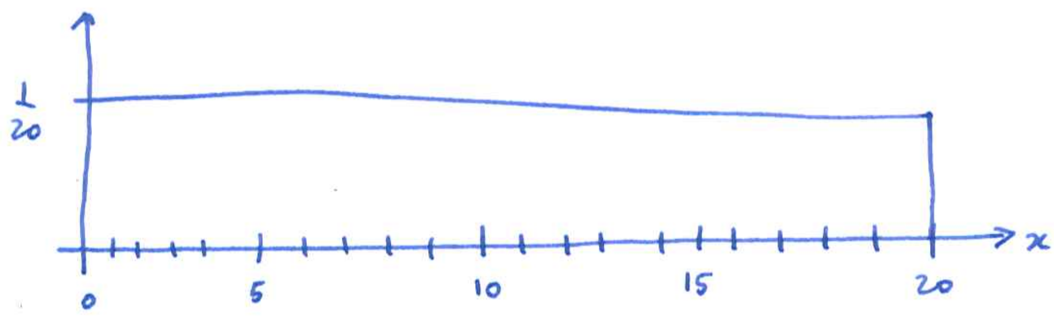
Area of rectangle = base · height

base: $2-0$

height is $\frac{1}{20}$

So the area we want is $P(0<X<2) = 2 \cdot \frac{1}{20} = \frac{1}{10} = 0.10$.

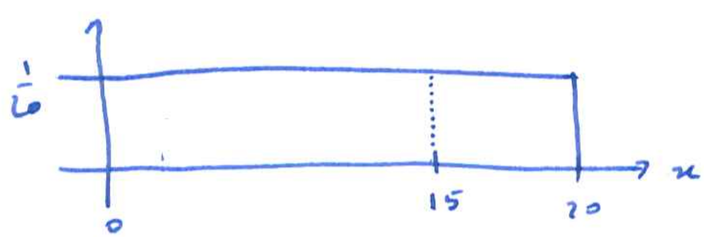
IN GROUPS: For $X \sim \text{unif}(0, 20)$, find $P(5 < X < 15)$.



Note that because $15 - 5 = 10$, the rectangle has base 10.
Height remains $\frac{1}{20}$, so area = base · height = $10 \cdot \frac{1}{20} = \frac{1}{2} = 0.5$.
Therefore, $P(5 < X < 15) = 0.5 = 50\%$.

The prob. that X is btwn. 5 and 15 is 50%.

Question: If $X \sim \text{unif}(0, 20)$, what is $P(X = 15)$?



Area of a line is exactly zero!

So $P(X = 15) = 0$.

In fact, for <u>ANY</u> continuous r.v. X (not just for uniform ones), $P(X = x)$, for some fixed x, is <u>ZERO</u>.

<u>Note</u>: That means that $P(X < a) = P(X \le a)$, and $P(X > b) = P(X \ge b)$, and so forth... for CONTINUOUS r.v. (but NOT for discrete r.v.).

So, if $X \sim unif(0, 20)$, and we know $P(5 < X < 15) = \frac{1}{2}$, then we know $P(5 \le X \le 15) = \frac{1}{2}$.

Also $P(5 \le X < 15) = \frac{1}{2}$ and $P(5 < X \le 15) = \frac{1}{2}$.