1, 2: Solve using elementary row operations on the eq'ns or augmented matrix. Follow the systematic elimination procedure described in the section.

1)
$$\begin{array}{l} x_1 + 5x_2 = 7 \\ -2x_1 - 7x_2 = -5 \end{array} \xrightarrow{2R_1 + R_2 \to R_2} \begin{array}{l} x_1 + 5x_2 = 7 \\ 3x_2 = 9 \end{array} \xrightarrow{\frac{1}{3}R_2 \to R_2} \begin{array}{l} x_1 + 5x_2 = 7 \\ x_2 = 3 \end{array} \xrightarrow{R_1 - 5R_2 \to R_1} \begin{array}{l} x_1 = -8 \\ x_2 = 3 \end{array}$$

Pivot

$$\left(\begin{array}{cc|c} \boxed{1} & 5 & 7 \\ -2 & -7 & -5 \end{array}\right) \longrightarrow \begin{array}{c} R_1 \\ 2R_1 + R_2 \end{array} \left(\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & \boxed{3} & 9 \end{array}\right) \to \begin{array}{c} R_1 \\ \frac{1}{3}R_2 \end{array} \left(\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & \boxed{1} & 3 \end{array}\right) \to \begin{array}{c} R_1 - 5R_2 \\ R_2 \end{array} \left(\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right)$$

Pivot Pivot

Both methods imply $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -8 \\ 3 \end{pmatrix}$. Check: $-8 + 5(3) \stackrel{?}{=} 7$ ✓

$-2(-8) - 7(3) \stackrel{?}{=} -5$ ✓

2)
$$\begin{array}{l} 2x_1 + 4x_2 = -4 \\ 5x_1 + 7x_2 = 11 \end{array} \quad \left(\begin{array}{cc|c} \boxed{2} & 4 & -4 \\ 5 & 7 & 11 \end{array}\right) \to \begin{array}{c} \frac{1}{2}R_1 \\ R_2 \end{array} \left(\begin{array}{cc|c} \boxed{1} & 2 & -2 \\ 5 & 7 & 11 \end{array}\right) \to \begin{array}{c} R_1 \\ R_2 - 5R_1 \end{array} \left(\begin{array}{cc|c} 1 & 2 & -2 \\ 0 & \boxed{-3} & 21 \end{array}\right) \to$$

Pivot Pivot Pivot

$$\to \begin{array}{c} R_1 \\ -\frac{1}{3}R_2 \end{array} \left(\begin{array}{cc|c} 1 & 2 & -2 \\ 0 & \boxed{1} & -7 \end{array}\right) \longrightarrow \begin{array}{c} R_1 - 2R_2 \\ R_2 \end{array} \left(\begin{array}{cc|c} 1 & 0 & 12 \\ 0 & 1 & -7 \end{array}\right)$$

Pivot

So, $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 12 \\ -7 \end{pmatrix}$. Check: $2(12) + 4(-7) \stackrel{?}{=} -4$ ✓

$5(12) + 7(-7) \stackrel{?}{=} 11$ ✓.

5, 6: Consider each matrix as the augmented matrix of a linear system. State in words the next two elementary row operations to perform.

5)
$$\left(\begin{array}{cccc|c} 1 & -4 & 5 & 0 & 7 \\ 0 & 1 & -3 & 0 & 6 \\ 0 & 0 & \boxed{1} & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right)$$

Pivot

Add 3(Row 3) + (Row 2) and store in Row 2.
Add 5(Row 3) + (Row 1) and store in row 1.

6)
$$\left(\begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & \boxed{1} & 2 & -3 \\ 0 & 0 & 3 & 1 & 6 \end{array}\right)$$

Pivot

Add -3(Row 3) + (Row 4) and store in row 4.
(The above gives R4: $0 \ 0 \ 0 \ -5 \mid 15$)
Store $-\frac{1}{5}$(Row 4) in Row 4.

17) Do the lines $x_1 - 4x_2 = 1$, $2x_1 - x_2 = -3$, and $-x_1 - 3x_2 = 4$ have a common point of intersec'n?

They intersect only when we can find x_1 and x_2 to solve the system:

$$\left\{\begin{array}{l} x_1 - 4x_2 = 1 \\ 2x_1 - x_2 = -3 \\ -x_1 - 3x_2 = 4 \end{array}\right., \text{ i.e., } \begin{pmatrix} 1 & -4 \\ 2 & -1 \\ -1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \to \left(\begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{array}\right) \to \begin{array}{c} R_1 \\ R_2 - 2R_1 \\ R_3 + R_1 \end{array} \left(\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{array}\right) \to$$

$$\to \begin{array}{c} R_1 \\ \frac{1}{7}R_2 \\ R_2 + R_3 \end{array} \left(\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{array}\right).$$

This system is consistent (has a sol'n), since the matrix row-reduces to a triangular one. So, YES, an intersec'n point exists.

18) Do the three planes $x_1 + 2x_2 + x_3 = 4$, $x_2 - x_3 = 1$, and $x_1 + 3x_2 = 0$ have at least one point of intersection?

Only if $\begin{cases} x_1 + 2x_2 + x_3 = 4 \\ \quad x_2 - x_3 = 1 \\ x_1 + 3x_2 \quad = 0 \end{cases}$ has at least one sol'n. Check by row-reducin:

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & 0 \end{array}\right) \to \begin{array}{c} R_1 \\ R_2 \\ R_3 - R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -4 \end{array}\right) \to \begin{array}{c} R_1 \\ R_2 \\ R_3 - R_2 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -5 \end{array}\right)$$

Now, Row 3 implies $0x_1 + 0x_2 + 0x_3 = -5$, but no choice of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ makes this true! $(0 \neq -5.)$ So, no — no points of intersection.

23, 24: Mark T/F & justify answer.

23) (a) Every elem. row operation is reversible.
True; see p. 7, last full paragraph.

(b) A 5×6 matrix has 6 rows.
False; it has 5 rows. (An $m \times n$ matrix has m rows, n cols.)

(c) The sol'n set of a linear system involving variables $(x_1, \dots, x_m)$ is a list of numbers $(s_1, \dots, s_m)$ tht. make each eq'n in the system true when $s_1, \dots, s_m$ are substituted, resp., for $x_1, \dots, x_m$.
False; this is a solution. The solution set is the set of all possible sol'ns.

(d) Two fundamental questions abt. a linear system involve existence & uniqueness.
True; see blue box, p. 8

24) (a) Elem. row operat'ns on an augmented matrix never change the sol'n set of the associated linear system. True; see p. 8, first sentence.

(b) Two matrices are row-equivalent if they have the same # of rows.
False; see def'n on p. 7, and e.g., $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ isn't row-equiv. to $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$.

(c) An inconsistent system has more than one sol'n.
False; see def'n, p. 4 which states a system is inconsistent when it has no sol'n.

(d) Two linear systems are equivalent if they have the same solution set.
True; see def'n, p. 3.

Examples of row reduction. CONF. 1

Ex. 3, p. 17.

1 ME

PIVOT ELT. IS 0 ... this is a problem

$$\left[\begin{array}{ccccc|c} \boxed{0} & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{array}\right]$$

PIVOT COLUMN IS 1^{st} COL.

$$\sim \begin{array}{l} \left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{array}\right] \end{array} \begin{array}{l} (R3) \\ (R2) \\ (R1) \end{array} \sim \left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{array}\right] \begin{array}{l} (R1) \\ (R2)-(R1) \\ (R3) \end{array} \sim$$

$$\sim \left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 3 & -6 & 6 & 2 & -5 \end{array}\right] \begin{array}{l} (R1) \\ \frac{1}{2}(R2) \\ (R3) \end{array} \sim \left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & -1 & 4 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ (R3)-3(R2) \end{array} \sim$$

ROW-ECHELON FORM:
(1) Zero-rows all bubbled down to bottom
(2) Entries below each pivot are zero
(3) Leading entry of the row is to the (R) of the l.e. of row above it.

$$\sim \left[\begin{array}{ccccc|c} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{array}\right] \begin{array}{l} \frac{1}{3}(R1) \\ (R2) \\ -(R3) \end{array} \sim \left[\begin{array}{ccccc|c} 1 & -3 & 4 & -3 & 0 & 13 \\ 0 & 1 & -2 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{array}\right] \begin{array}{l} (R1)-2(R3) \\ (R2)-(R3) \\ (R3) \end{array}$$

~ ea pivot

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 0 & 16 \\ 0 & 1 & -2 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{array}\right] \begin{array}{l} (R1)+3(R2) \\ (R2) \\ (R3) \end{array}$$

CONSISTENT.

SOL.

$$\begin{cases} x_3 = r, \quad x_4 = s. \\ x_1 = 16 + 2r - 3s \\ x_2 = 1 + 2r - 2s \\ x_5 = -4 \end{cases}$$

RREF:
(1) Pivot elts. all 1
(2) Each pivot is the only nonzero elt in its col.

Ex. 17, p. 11.

4 B TOGETHER

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{array}\right] \begin{array}{l} (R1) \\ (R2)-2(R1) \\ (R3)+(R1) \end{array} \sim \left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} (R1) \\ 1/7(R2) \\ (R3)+(R2) \end{array} \sim \underbrace{\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} (R1)+4(R2) \\ (R2) \\ (R3) \end{array}}_{\text{RREF}}$$

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{array}\right] \text{ REF}$$

$$\text{sol.} \begin{cases} x_1 = -2 \\ x_2 = -5/7 \end{cases}$$

Made-up.

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & -6 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & -5 \end{array}\right] \begin{array}{l} (R1) \\ (R2)-2(R1) \\ (R3)+(R1) \end{array} \sim \left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & -10 \end{array}\right]$$

INCONSISTENT

CAN BE ANYTHING BUT 4, SYS. W/B INCONSISTENT.

Practice Exercises: p. 11-12, # 1-23 odd, p. 25-26 # 1-6 all, # 7-31 (odd), p. 47-48 # 1-21 odd, p. 55, # 1-11 odd

p. 11-12

1-8: Solve each system by using elementary row operations on the eq'ns or augmented matrix.

1) $\begin{aligned} x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5 \end{aligned} \sim \begin{aligned} x_1 + 5x_2 &= 7 && (E1) \\ 3x_2 &= 9 && (E2 + 2E1) \end{aligned}$

Then $x_2 = 3$, $x_1 = 7 - 5(3) = -8$.

3) Find the point (x_1, x_2) lying on the line $x_1 + 5x_2 = 7$ and on the line $x_1 - 2x_2 = -2$.

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ x_1 - 2x_2 &= -2 \end{aligned} \to \left[\begin{array}{cc|c} 1 & 5 & 7 \\ 1 & -2 & -2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 9 & 9 \end{array}\right] \begin{array}{l} (R1) \\ (R1-R2) \end{array}$$

Then by R2, $9x_2 = 9 \Rightarrow x_2 = 1$
and by R1, $x_1 = 7 - 5x_2 = 7 - 5 = 2$

5) State the next two elementary row operations that should be performed in solving.

$$\left[\begin{array}{cccc|c} 1 & -4 & 5 & 0 & 7 \\ 0 & 1 & -3 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right]$$

$(R2) \leftarrow 3(R3) + (R2)$ } Add 3 times row 3 to row 2, store result in row 2.

$(R1) \leftarrow -5(R3) + (R1)$ } Add -5 times row 3 to row 1, store in row 1.

7,9: Continue to solve, describe sol'n.

7) $\left[\begin{array}{ccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$ no solutions (row 3)

going from REF to RREF

Pill

9) $\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ (R3) \\ (R4)\cdot\frac{1}{2} \end{array} \sim$

$\sim \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ (R3)+3(R4) \\ (R4) \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} (R1) \\ (R2)+3(R3) \\ (R3) \\ (R4) \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} (R1)+(R2) \\ (R2) \\ (R3) \\ (R4) \end{array}$ –

The sol'n is $(4, 8, 5, 2)$.

11,13: Solve the system.

11) $\begin{aligned} x_2 + 4x_3 &= -5 \\ x_1 + 3x_2 + 5x_3 &= -2 \\ 3x_1 + 7x_2 + 7x_3 &= 6 \end{aligned} \to \left[\begin{array}{ccc|c} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 3 & 7 & 7 & 6 \end{array}\right] \begin{array}{l} (R2) \\ (R1) \\ (R3) \end{array} \sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 2 & 8 & -12 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ 3(R1)-R3 \end{array} \sim$

THEM

$\sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 1 & 4 & -10 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ \frac{1}{2}(R3) \end{array} \sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & -5 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ (R3)-(R2) \end{array}$

NO SOL'NS (R3)

Pill

13) $\begin{aligned} x_1 \quad\quad - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned} \to \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right] \begin{array}{l} (R1) \\ (R2)-2(R1) \\ (R3) \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array}\right] \begin{array}{l} (R1) \\ (R3) \\ (R2) \end{array} \sim$

$\sim \underbrace{\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array}\right]}_{\text{REF}} \begin{array}{l} (R1) \\ (R2) \\ (R3)-2(R2) \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ \frac{1}{5}(R3) \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{l} (R1)+3(R3) \\ (R2)-5(R3) \\ (R3) \end{array}$

Sol'n: $\begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$

Reading off a solution set from a matrix in RREF.

p.25

3

1) (a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] \rightarrow$ sol'n is $\begin{array}{l} 1x_1 = 0 \\ 1x_2 = 0 \\ 1x_3 = 1 \end{array} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 1 \end{cases}$

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \rightarrow$ NO SOL'N (inconsistent)

2) (a) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow$ $\begin{array}{l} x_1 + x_2 = 1 \\ x_3 = 1 \end{array}$ $\Rightarrow$ sol'n is $\begin{cases} x_1 = 1 - b \\ x_2 = r, r \in \mathbb{R} \\ x_3 = 1 \end{cases}$

let $x_2 = r$, $r \in \mathbb{R}$

(d) $\left[\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow$ NO SOL'N

Matrix Multiplication.

5 To multiply two matrices A and B...

$$[\ A\ ]\cdot[\ B\ ] = [\ C\ ]$$

$m \times n$ $\quad n \times k \quad$ $m \times k$

MUST MATCH

DIM. OF SOLUTION...

the (i,j) entry of AB is i^{th} ROW of A dotted w. j^{th} COL of B.

Eg.,

6

$$\overset{A}{\begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}} \cdot \overset{B}{\begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}} = \overset{AB}{\begin{bmatrix} x & x & x \\ x & x & x \end{bmatrix}}$$

$2 \times 2 \qquad 2 \times 3$

$$AB_{11} = [2 \ \ 3] \cdot \begin{bmatrix} 4 \\ 1 \end{bmatrix} = 2(4) + 3(1) = 11 \qquad \begin{bmatrix} 11 & x & x \\ x & x & x \end{bmatrix}$$

$$AB_{21} = [1 \ -5] \cdot \begin{bmatrix} 4 \\ 1 \end{bmatrix} = 1(4) - 5(1) = -1 \qquad \begin{bmatrix} 11 & x & x \\ -1 & x & x \end{bmatrix}$$

$$AB_{12} = [2 \ \ 3] \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix} = 2(3) + 3(-2) = 0 \qquad \begin{bmatrix} 11 & 0 & x \\ -1 & x & x \end{bmatrix}$$

$$AB_{22} = [1 \ -5] \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix} = 1(3) + 5(2) = 13 \qquad \begin{bmatrix} 11 & 0 & x \\ -1 & 13 & x \end{bmatrix}$$

$$AB_{13} = [2 \ \ 3] \cdot \begin{bmatrix} 6 \\ 3 \end{bmatrix} = 2(6) + 3(3) = 27 \qquad \begin{bmatrix} 11 & 0 & 27 \\ -1 & 13 & x \end{bmatrix}$$

$$AB_{23} = [1 \ -5] \begin{bmatrix} 6 \\ 3 \end{bmatrix} = 1(6) - 5(3) = -9 \qquad \begin{bmatrix} 11 & 0 & 27 \\ -1 & 13 & 9 \end{bmatrix}.$$

$$AB = \begin{bmatrix} 11 & 0 & 27 \\ -1 & 13 & 9 \end{bmatrix}$$

15) Determine whether the systems are consistent.

$$\begin{aligned} x_1 \quad + 3x_3 \quad &= 2 \\ x_2 \quad - 3x_4 &= 3 \\ -2x_2 + 3x_3 + 2x_4 &= 1 \\ 3x_1 \quad + 7x_4 &= -5 \end{aligned} \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ (R3) \\ (R4)-3(R1) \end{array} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ (R3)+2(R2) \\ (R4) \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & 0 & -5 & 10 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ (R3) \\ (R4)+3(R3) \end{array}$$

CONSISTENT.

17) Do the three lines $x_1 - 4x_2 = 1$, $2x_1 - x_2 = -3$, and $-x_1 - 3x_2 = 4$ have a common point of intersec'n? Explain.

The point of intersec'n will satisfy the system:

$$\begin{aligned} x_1 - 4x_2 &= 1 \\ 2x_1 - x_2 &= -3 \\ -x_1 - 3x_2 &= 4 \end{aligned} \rightarrow \left[\begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{array}\right] \begin{array}{l} (R1) \\ (R2)-2(R1) \\ (R3)+(R1) \end{array} \sim \left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} (R1) \\ (R2) \\ (R3)+(R2) \end{array}$$

It has a unique sol'n, as the system is consistent.

---

p. 47, 1-21 odd.

1) NO :(

3) $$\underbrace{\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix}}_{3\times 2} \cdot \underbrace{\begin{bmatrix} 2 \\ -3 \end{bmatrix}}_{2\times 1} = \begin{bmatrix} [6\ 5]\cdot\begin{bmatrix} 2 \\ -3 \end{bmatrix} \\ [-4\ -3]\cdot\begin{bmatrix} 2 \\ -3 \end{bmatrix} \\ [7\ 6]\cdot\begin{bmatrix} 2 \\ -3 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 6(2)-5(3) \\ -4(2)+3(3) \\ 7(2)-6(3) \end{bmatrix} = \begin{bmatrix} 12-15 \\ -8+9 \\ 14-18 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}$$

7

3×1

5) $$\underbrace{\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}}_{\text{MAT. EQ'N}} \sim 5\begin{bmatrix} 5 \\ -2 \end{bmatrix} - \begin{bmatrix} 1 \\ -7 \end{bmatrix} + 3\begin{bmatrix} -8 \\ 3 \end{bmatrix} - 2\begin{bmatrix} 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$$

7) KEY.

$$x_1\begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2\begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ 7 \end{bmatrix} \sim \begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ 7 \end{bmatrix}$$