Oct. 28 (lec. 23)

Housekeeping: • Canvas HW 11:59 p.m. today

• HW 5 due Wednesday in class

Last time: • Derivatives

• Area btwn. curve & axis

• Arc Length

Today: • Arc length practice

• Areas of surfaces of revol'n.

Warm-up :

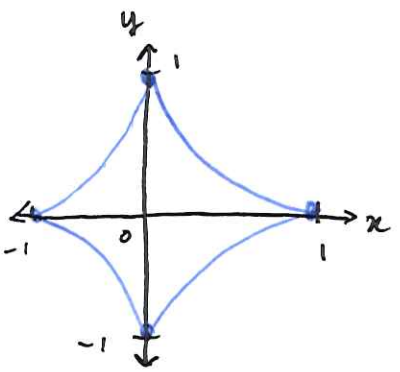
Find the length of the astroid $\begin{cases} x = \cos^3 t \\ y = \sin^3 t \\ 0 \le t \le 2\pi \end{cases}$

Check : ✓ $\frac{dx}{dt}$ & $\frac{dy}{dt}$ are cts. on $[0, 2\pi]$

✓ $\frac{dx}{dt}$ & $\frac{dy}{dt}$ are not simultaneously zero on $[0, 2\pi]$

✓ Astroid is traversed exactly once as t inc. from 0 to 2π

$\frac{dx}{dt} = -3\cos^2 t \sin t$, $\frac{dy}{dt} = 3\sin^2 t \cos t$.



Note: $\frac{dx}{dt}$ & $\frac{dy}{dt}$ are simultaneously zero when $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$

If we restrict the parameter interval to $(0, \frac{\pi}{2})$, then the derivs. of x & y remain cts, and in add'n, are not simultaneously zero.

Exploiting symmetry , $L = 4\int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= 4\int_0^{\pi/2} \sqrt{9\cos^4 \sin^2 t + 9\sin^4 \cos^2 t}\, dt$

$= 12\int_0^{\pi/2} \sqrt{\sin^2 t \cos^4 t + \sin^4 t \cos^2 t}\, dt$

$$= 12 \int_0^{\pi/2} \underbrace{\sin t \cos t}_{\substack{\text{both positive} \\ \text{on } (0, \pi/2)}} \sqrt{\underbrace{\cos^2 t + \sin^2 t}_{=1}} \, dt$$

$$= 12 \int_0^{\pi/2} \sin t \cos t \, dt$$

let $u := \sin t$ then $u(0) = 0$

$du = \cos t \, dt$ $u(\frac{\pi}{2}) = 1$

$$= 12 \int_0^1 u \, du = \frac{12}{2} u^2 \Big|_0^1 = \boxed{6 \text{ units}}$$

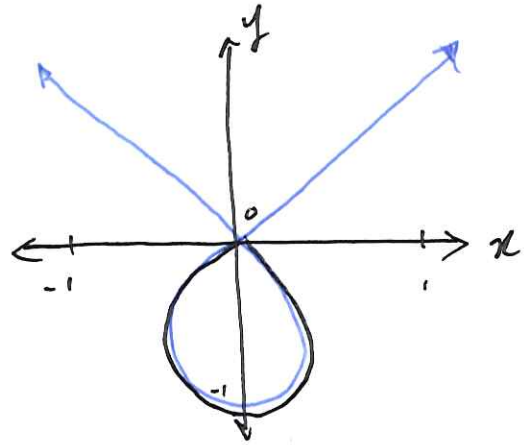
Find the perimeter of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \qquad (a \neq 0, \ b \neq 0)$$

Could choose parametrization $\begin{cases} x = a \sin(t) \\ y = b \cos(t) \\ 0 \le t \le 2\pi \end{cases}$.

Example 29c.

$$\begin{cases} x = t(t^2-1), \\ y = t^2-1 \\ -1 \le t \le 1 \end{cases}$$



Find the arc length of the teardrop — we would find where the graph crosses itself.

Want to find t_1 and t_2 such that

$$x(t_1) = x(t_2) \iff \boxed{t_1(t_1^2-1) = t_2(t_2^2-1)} \quad (1)$$

and

$$y(t_1) = y(t_2) \iff \boxed{t_1^2-1 = t_2^2-1} \quad (2)$$

$$\left.\begin{aligned} x(-1) &= -1((-1)^2-1) = -1(0) = 0 \\ x(1) &= 1(1^2-1) = 1(0) = 0 \end{aligned}\right\} \text{same}$$

$$\left.\begin{aligned} y(-1) &= (-1)^2-1 = 0 \\ y(1) &= 1^2-1 = 0 \end{aligned}\right\} \text{same}$$

So, indeed, the graph crosses itself at $(x,y) = (0,0)$, corr. to $t = \pm 1$.

Check: $\frac{dx}{dt}$, $\frac{dy}{dt}$ (1) cts, (2) not sim. 0, and (3) graph is traced once

$$L = \int_{-1}^{1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_{-1}^{1} \sqrt{9t^4 - 2t^2 + 1}\, dt$$