Lec. 2~~3~~4 - Oct. 31, 2016.

Housekeeping: • HW 5 due W in class

• Canvas HW due tonight (?)

Last time: Derivatives

Arc length

Today: Surfaces of revol'n.

If a smooth curve (i.e., ctsly diff'ble) $x = f(t)$, $y = g(t)$, $t \in [a,b]$, is traversed exactly once as t inc. from a to b, then the areas of the surfaces generated by the revol'n of the curve abt. the coordinate axes is:

(1) Revol'n abt. x-axis ($y \geq 0$):

$$S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

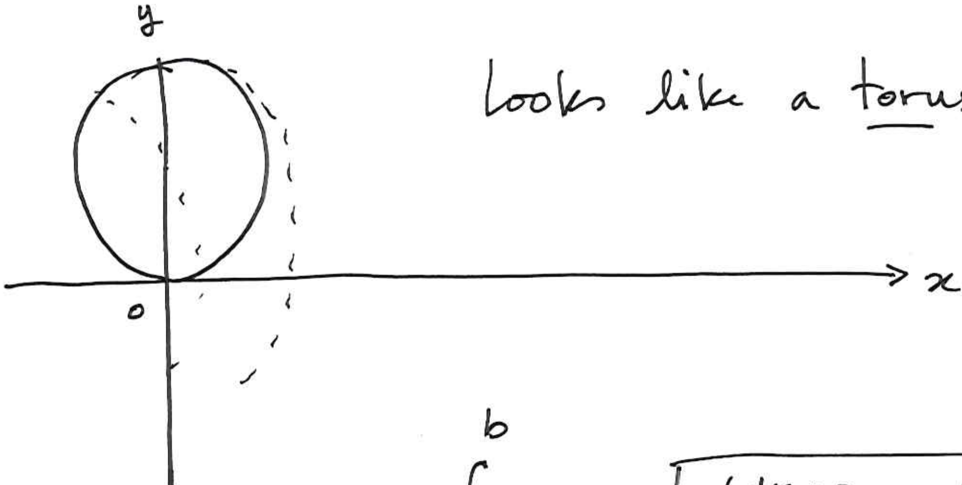
(2) Revol'n abt. y-axis ($x \geq 0$):

$$S = \int_a^b 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Example (1) Unit circle : center $(0,1)$

$$\begin{cases} x = \cos(t) \\ y = 1 + \sin(t) \\ t \in [0, 2\pi] \end{cases}$$

Find the area of surface swept out by revolving around x-axis.



Looks like a torus.

$$S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dx}{dt} = \frac{d}{dt}[\cos t] = -\sin t$$

$$\frac{dy}{dt} = \frac{d}{dt}[1 + \sin t] = \cos t$$

$$S = \int_0^{2\pi} 2\pi (1 + \sin t) \sqrt{\cos^2 t + (-\sin t)^2}\, dt$$

$$= \int_0^{2\pi} 2\pi (1 + \sin t) \sqrt{\sin^2 t + \cos^2 t}\, dt$$

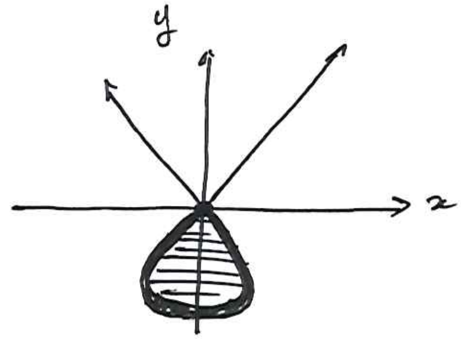
$$= \int_0^{2\pi} 2\pi (1 + \sin t)\, dt$$

$$= 2\pi t - 2\pi \cos t \Big|_0^{2\pi} = 4\pi^2.$$

$$= 2\pi \cdot 2\pi - 2\pi \cos(2\pi) - 2\pi(0) + 2\pi \cos(0)$$

Example ②.

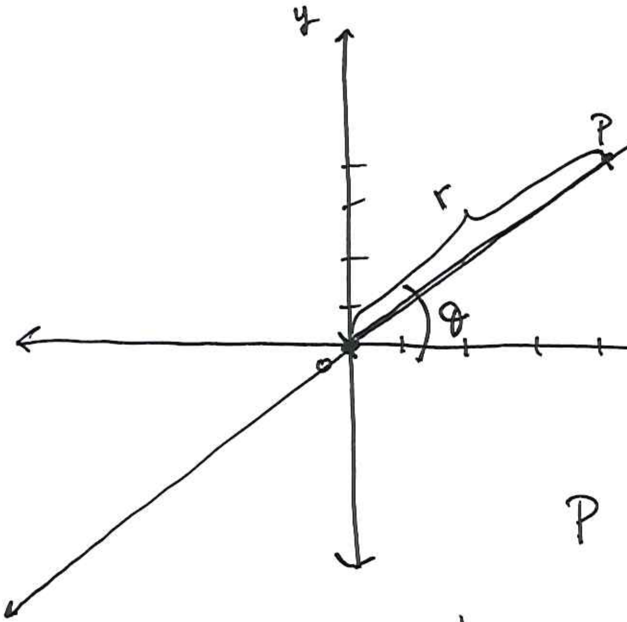
$$\begin{cases} x = t(t^2-1) \\ y = t^2-1 \\ t \in [-1, 1] \end{cases}$$



Revolve around x-axis — what is the surf area of solid of revol'n?

Check: Smooth? Traced out once?

Polar coordinates vs. Cartesian



Cartesian

(x,y) is given as a label for P by determining that P lies x many units to the R of O and y many units upward from O.

Polar

P could also be labelled as (r, θ),

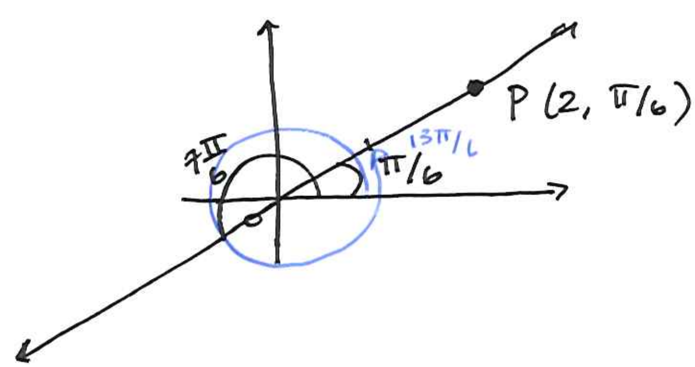
because to get from the origin O to the pt. P, you travel r many units along the line at angle θ w. the positive x-axis.

For example, 3

$P\left(2, \frac{\pi}{6}\right)$

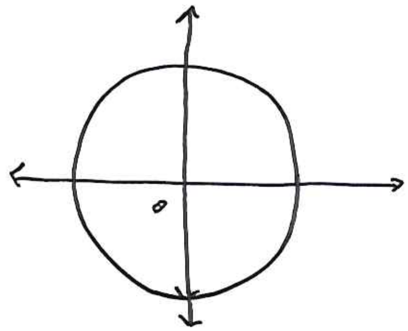
$P\left(2, \frac{13\pi}{6}\right)$

$P\left(-2, 7\pi/6\right)$



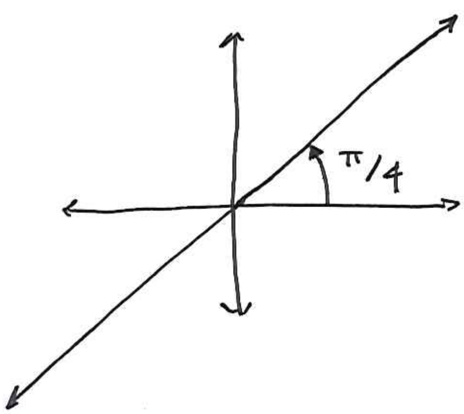
Eq'ns in Polar Coordinates.

Example 4 Circle ctr. at origin:



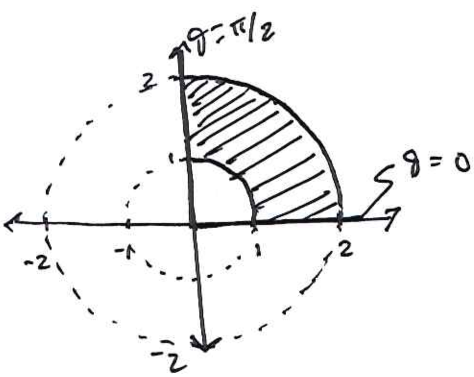
$r = 1$ describes all points (r, θ) on the unit circle.

Example 5 Line through origin; at angle $\pi/4$



$\theta = \frac{\pi}{4}$.

Example 6 $\{(r,\theta) : 1 \le r \le 2 \text{ and } 0 \le \theta \le \frac{\pi}{2}\}$



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Ex. 7

$$\left\{ (r, \theta) : \frac{2\pi}{3} \le \theta \le \frac{5\pi}{6} \right\}$$