

Homework 4: Due in class October 26

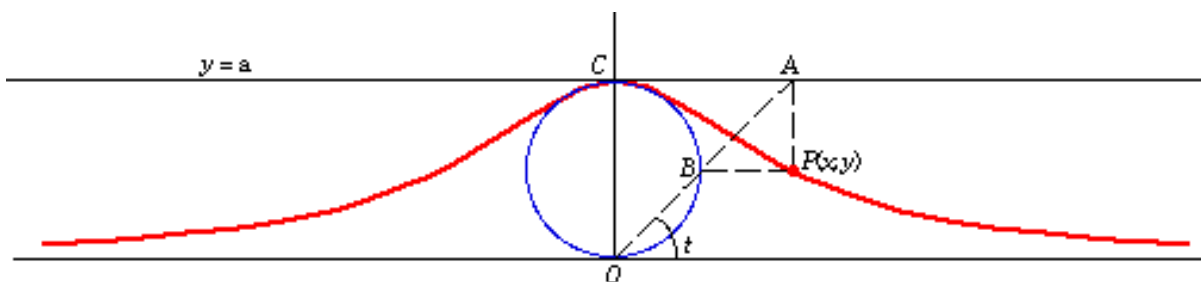
Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words¹, and if you elect to use symbols, you must use them properly. In particular, please avoid the use of the “running equals sign”, as this is an abuse of notation and is unacceptable: http://www.wikiwand.com/en/Equals_sign#/Incorrect_usage. Write your solutions so that a student one course behind you in the sequence would understand them.

Problem 1. Find a parametrization for the following curves:

- (a) [2 points] The line segment with endpoints $(-1, -3)$ and $(3, 2)$.
- (b) [2 points] The ray (half line) with initial point $(2, 3)$ that also passes through the point $(-1, 1)$.
- (c) [3 points] The lower half of the parabola $x - 2 = 2y^2$.
- (d) [3 points] The left-hand half of the parabola $y = x^2 + 2x$.

Problem 2. [10 points] The bell-shaped witches of Maria Agnesi² form a set of curves; we will construct one of them in the following way. Starting with a circle of radius 1 and centered at the point $(0, 1)$, choose a point A on the line $y = 2$ and connect it to the origin with a line segment. Call the point where the segment crosses the circle B , and let P be the point where the vertical line through A crosses the horizontal line through B . The witch³ is the curve traced by P as A moves along the line $y = 2$. (In the picture below, take $a = 2$.)



Find parametric equations and a parameter interval for the witch by expressing the coordinates of P in terms of t , the radian measure of the angle that the segment OA makes with the positive x -axis. It may help if you use the following equalities (which you do not have to prove):

$$x = AC, \quad y = 2 - AB \sin t, \quad AB \cdot OA = (AC)^2.$$

Remember that on this and all homework problems, **you must show your work**. If you write down the parametric equations with no explanation, you will get zero credit.

¹See a list of mathematical symbols and their meanings here: http://en.wikipedia.org/wiki/List_of_mathematical_symbols

²Agnesi was an interesting figure in mathematics; the curious can read a little about her life here: https://en.wikipedia.org/wiki/Maria_Gaetana_Agnesi.

³For a .gif of this construction process, see <http://mathworld.wolfram.com/images/gifs/WitchOfAgnesi.gif>.