L15: March 21, 2017.

Last time: Space Mapping

Modelling free fall

QUESTIONS?

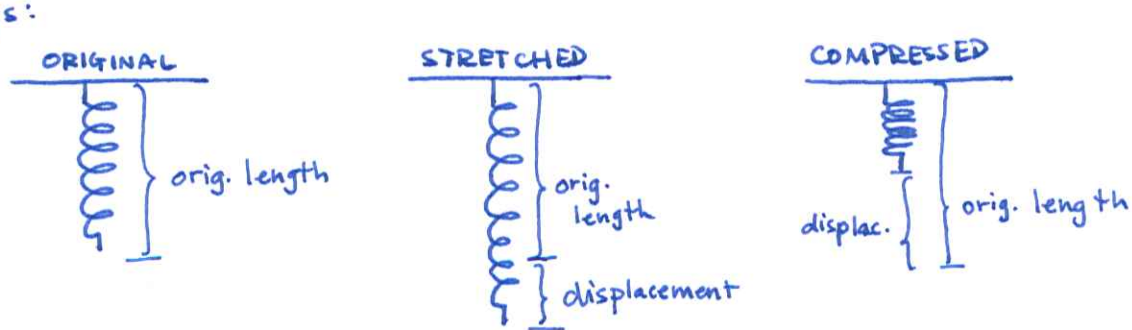
This time: Module 3.2: Modelling bungee jumping

Bungee jumping has an interesting history (read p. 78, Introduc'n to Module 3.2).

Bungee cords act similarly to springs — so, can use a model similar to a spring model.

(Good for us, as the physics of springs is well known...)

Springs:



(Monday was the first day of spring & the vernal equinox, btw...)

When you stretch a spring, a force tries to restore the spring's length to the original length (same happens when you compress a spring).

When you stretch (or compress) a spring by a greater displacement, the restoring force is stronger.

(This is how a BOWFLEX® machine works — setting a certain resistance entails pre-elongating a spring to the point where the spring's restoring force is exactly the desired amount.)

i.e., can spring back fully

Hooke's Law pertains to springs that are elastic, and states: within the elastic limit of a spring, the magnitude of force needed to produce a certain displacement is directly proportional to the displacement itself:

$$F = -ks, \quad \text{where}$$

F is the applied force; $[F] = N = kg \cdot m/s^2$

s is the displacement; $[s] = m$

k is the spring's constant (dep. on spring!); $[k] = kg/s^2$.

Quick Review: What force is required to stretch a spring 0.1 m from its original length, if that spring's constant is 5 kg/s^2 ?

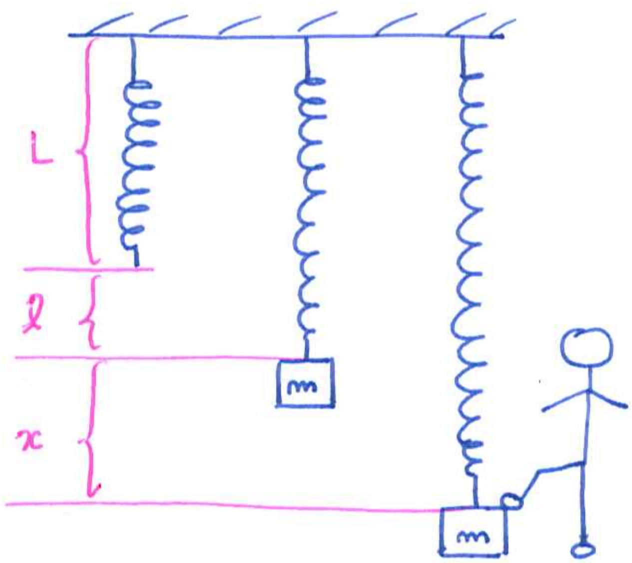
$$F = -ks = (-0.1\,m)(5\,kg/s^2) = -0.5\,\frac{kg \cdot m}{s^2} = -0.5\,N.$$

Add'l Question: If a 30-lb. weight stretches a spring by 2ft. from its equilibrium position, then what is the spring's constant ?

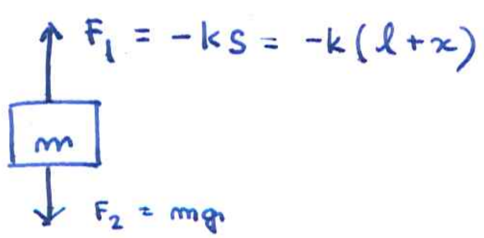
$$F = -ks \iff -30\,lb = -k \cdot 2ft \iff k = \frac{30\,lb.}{2ft} = 15\,\frac{lb}{ft}.$$

Quick Review 2(a).

$$\text{length} = L + \underbrace{l + x}_{S}$$

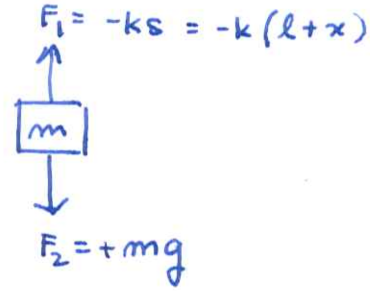


QR 2(b).



QR 2(c). time, spring const., mass of jumper, height of the cliff, length of unstretched bungee cord, limits of elastic deformation of bungee cord, gravitat'n constant, weighted length of bungee cord.

Now... we know that the mass undergoes:



When the spring & mass are in equilibrium, they remain stationary – i.e., the forces balance:

$$-mg = -kl \quad , \text{ or } \quad kl = mg .$$

Therefore, at all other times (not necessarily in equilibrium), the mass undergoes the restoring force from the spring:

$$F_1 = -k(l+x) = -kl - kx$$

$$= -mg - kx .$$

and $F_2 = +mg$.

By Newton's 2nd law, the net force acting on mass is:

$$F := F_1 + F_2$$

and the acceleration of the mass is given by:

$$F = ma .$$

$$F = F_1 + F_2 = -\cancel{mg} - kx + \cancel{mg}$$

$$F = -kx$$

or

$$F = -k\,x(t)$$, since x depends on time

Moreover, the acceleration is given by

$$F = ma,$$

or

$$-k\,x(t) = m\,a(t)$$ (reminder: accel. is also time-dependent)

Recall that $x''(t) = a(t)$.

So,

$$-\frac{k}{m}x(t) = x''(t).$$

i.e.,

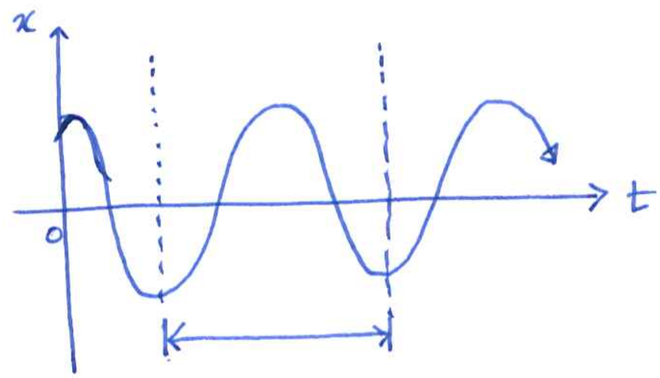
$$x''(t) + \frac{k}{m}x(t) = 0.$$

This is the ODE that governs the motion of the mass.

The ODE $x''(t) + \frac{k}{m} x(t) = 0$ is solved by

$$x(t) = c_1 \cos\left(\sqrt{\tfrac{k}{m}}\, t\right) + c_2 \sin\left(\sqrt{\tfrac{k}{m}}\, t\right).$$

Sol'ns look like:



The period of the waves is $\frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{m}{k}}$.

When k increases, the "wavelength" decreases —
when m decreases ——— " ———

This is Simple Harmonic Motion (SHM) - ignoring fric'n / damping.