L18 : March 30, 2017. 1

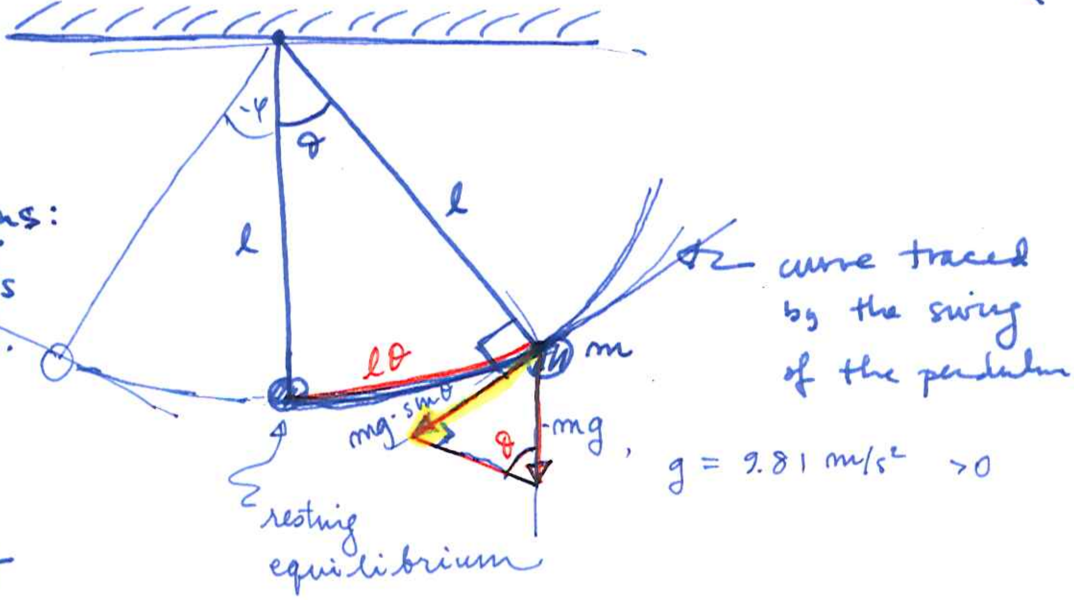
Housekeeping.

- A15/16 was due Tuesday night (spring model)
- A17 (bungee model) code due tonight, 11:59 p.m.
- A18 – bungee report due Tuesday.

Last time: converting springs to bungee cords

This time: Pendulum swings

Pendulum:

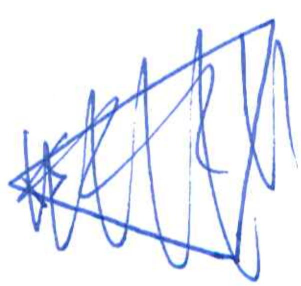


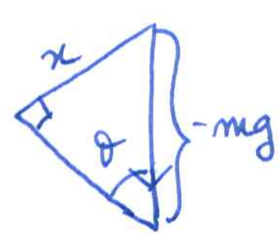
- Simplifying assump'ns:
  - mass of the bob is concentrated at a pt.
  - Stiff rod has no mass
  - Fric'n doesn't exist

Notes: The force of gravity acts on the bob, but the bob cannot fall straight down — can only fall in the direc'n of its swing.

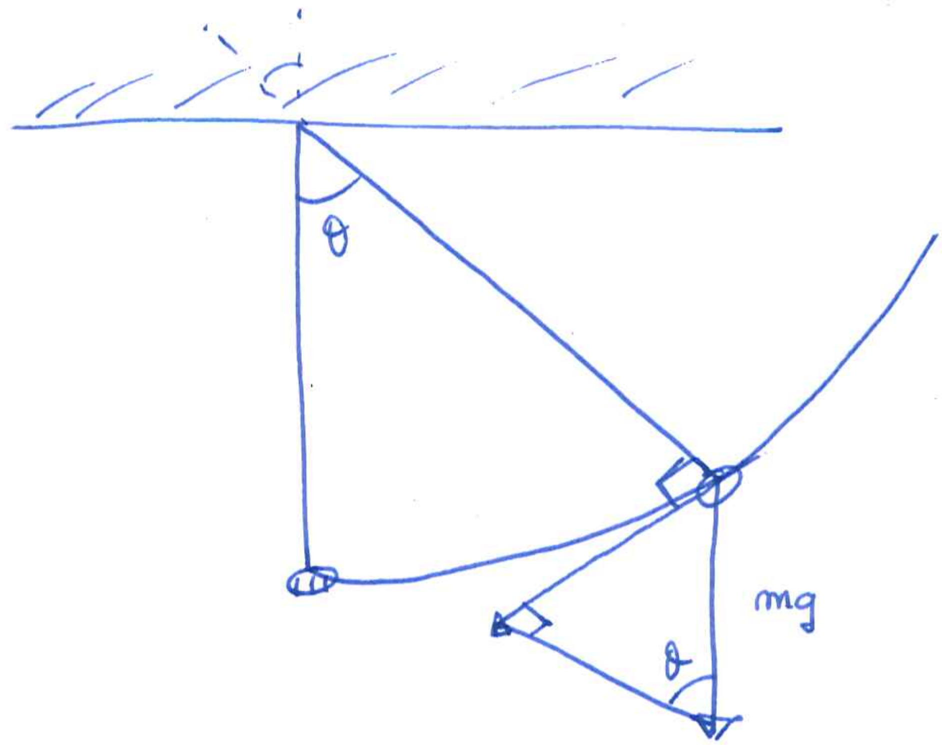
That is, gravity acts on the bob via the projection of the gravitat'l force vector on the vector tangent to the curve of the swing.

The magnitude of the vector projec'n is $m \cdot g \cdot \sin(\theta)$

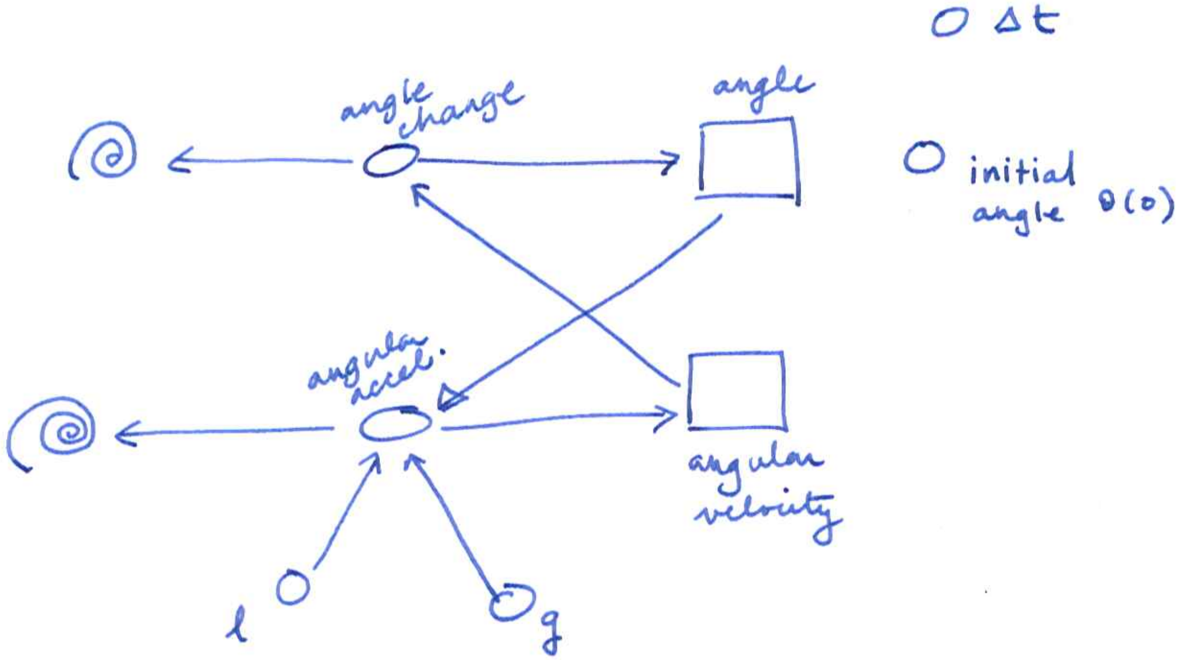




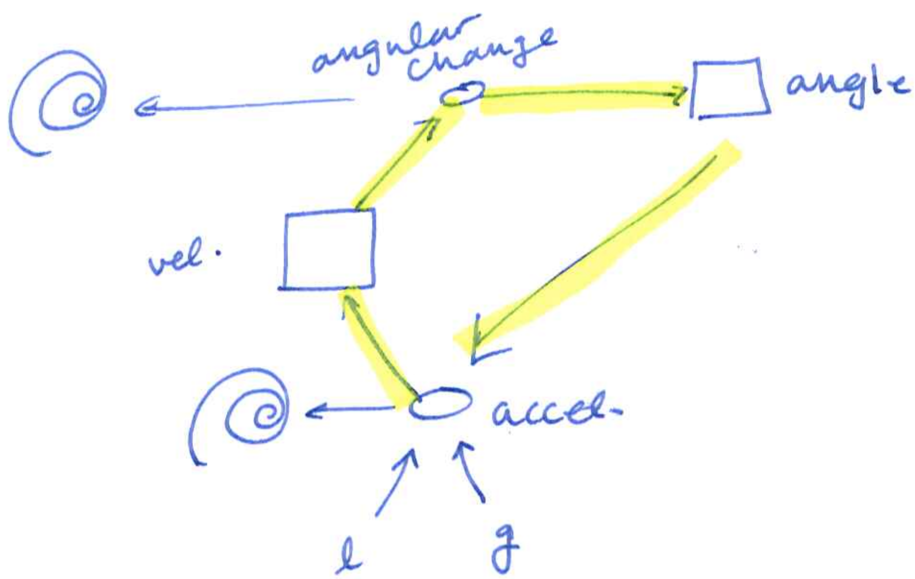
$$\sin\theta = \frac{x}{-mg} \Rightarrow x = -mg \cdot \sin\theta$$


θ
mg
θ

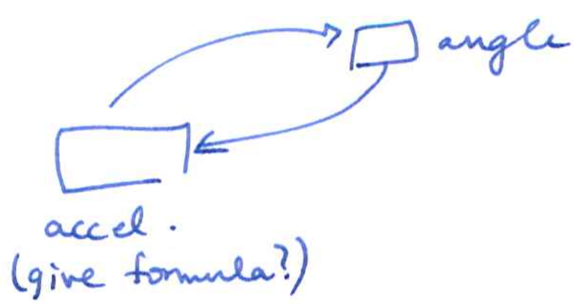
The model:



○ Δt
angle change
angle
○ initial angle θ(0)
angular accel.
angular velocity
l
g



angular change
angle
vel.
accel.
l
g



angle
accel.
(give formula?)

Consider the angular motion of the pendulum — according to a geometric formula, the arc length from the bob to resting equilibrium is:

$$\ell\theta \,.$$

So, the acceleration along the swing of the bob is the second derivative of this length w.r.t:

angular accel. $= \dfrac{d^2}{dt^2}\left[\ell\theta\right]$

$$a(t) = \ell \, \frac{d^2\theta}{dt^2} \,.$$

note: $\theta = \theta(t)$, but ℓ is constant in time.

On the other hand, by Newton 2:

force $=$ mass $\cdot\, a(t)$

$$-\cancel{m}\cdot g\cdot \sin\theta = \cancel{m}\cdot a(t)$$

$$a(t) = -g\cdot\sin\theta$$

$$-g\cdot\sin\theta = \ell\cdot\frac{d^2\theta}{dt^2} \,.$$

DIFF'L EQ'N FOR $\theta(t)$: $\boxed{\dfrac{d^2\theta}{dt^2} + \dfrac{g}{\ell}\sin\theta = 0 \,.}$

A solution set for this diff'l eq'n — which is a 2^{nd} order, nonlinear, homogeneous ODE — will have two free parameters.

These parameters can be solved for when two cond'ns are given — either a set of 2 initial cond'ns, or a set of 2 boundary cond'ns — e.g.,

$$\theta(0) = \frac{\pi}{4}, \quad \theta'(0) = 0 \quad \Big\} \text{ INITIAL COND'NS}$$

$$\theta(0) = \frac{\pi}{4}, \quad \theta(1) = 0 \quad \Big\} \text{ BDRY. COND'NS.}$$

QR1. (a) Determine variables & units in the metric system.

F [N], l [m], θ [rad], t [s], $\theta'(t)$ $\left[\frac{rad}{s}\right]$, $\theta''(t)$ $\left[\frac{rad}{s^2}\right]$

(b) Which of these variables is the rate of change of angular velocity?

$\theta''(t)$

(c) What is the flow into a stock/box variable of angular velocity?

$(\Delta t)\,\theta''(t)$

(d) Give the relevant ODE

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0.$$

(e) What is the flow into a stock/box variable of angle itself?

$(\Delta t)\,\theta'(t)$ $\quad \theta(t) = \arcsin\left(\frac{-l}{g}\,\theta''(t)\right)$.