L19: April 6, 2017.

Housekeeping: Choose a final project (HW1 topics or any "projects" problem from back of chapter)

Pendulum homework due Tuesday - written problems AND report on computer model

Last time: Pendulum model

This time: Friction as damping for pendulum model

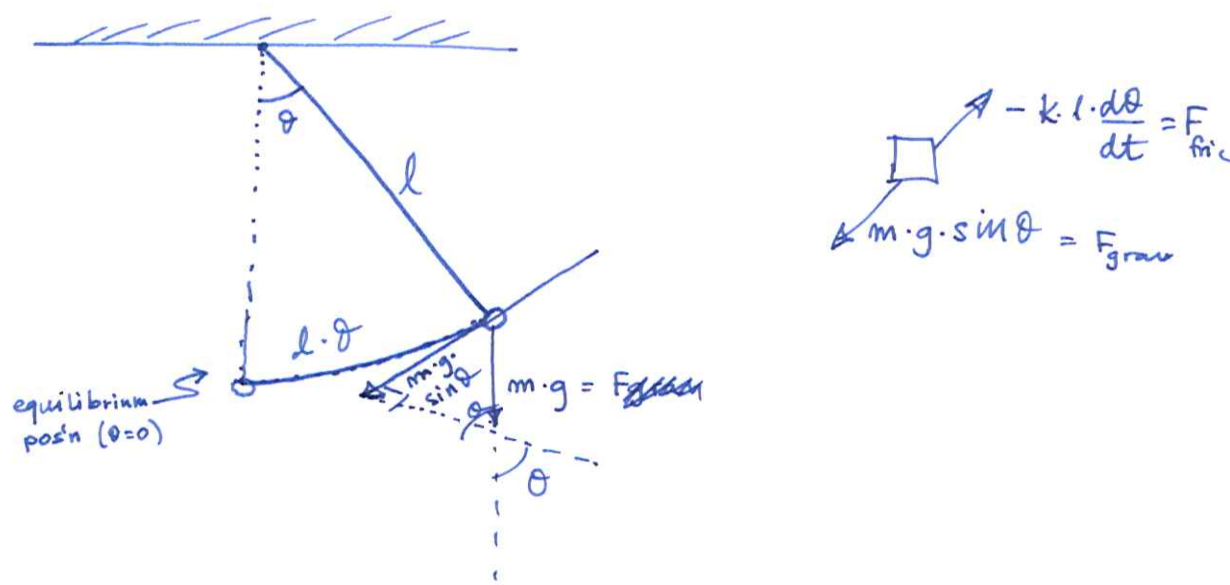
For a real pendulum, motion is damped by friction...

RECALL:

- Stokes' fric'n : very small object moving slowly through fluid ; $F = -kv$

- Newtonian fric'n : larger objects moving faster through fluid ; $F = \underbrace{0.5\,CDA}_{=0.65 \text{ for air at sea level}}\, v^2$

For the pendulum of a clock, we can safely use the simpler model of Stokes' friction — so we update the forces :



$$l \cdot m \cdot \frac{d^2\theta}{dt^2} = F_{net} = F_{grav} + F_{fric}$$

$$l \cdot m \cdot \frac{d^2\theta}{dt^2} = F_{net} = mg\sin\theta - k\frac{d\theta}{dt} \cdot l$$

$$\frac{d^2\theta}{dt^2} = \frac{g}{\ell}\sin\theta - \frac{k}{m}\frac{d\theta}{dt}$$

$$\boxed{\frac{d^2\theta}{dt^2} + \underline{\underline{\frac{k}{m}}}\frac{d\theta}{dt} - \frac{g}{\ell}\sin\theta = 0}$$

(p. 94 of text)

This governs pendulum motion subject to Stokes' friction.

$$\begin{cases} \theta(0) = \theta_0 & \text{initial angular displacement} \\ \left.\frac{d\theta}{dt}\right|_{t=0} = 0 & \text{initial velocity} \end{cases}$$

Assume $k = 6\pi \cdot D_v \cdot R$, $R = 0.01$ mm

$\left. D_v \right|_{\substack{40°F \\ 1\,ATM}} = 3.60$ $\left. D_v \right|_{\substack{70°F \\ \text{(std. Atm. pressure)}}} = 3.82$, $\left. D_v \right|_{\substack{60°F \\ 1\,ATM}} = 3.75$, $\left. D_v \right|_{\substack{50°F \\ 1\,ATM}} = 3.68$.

$(70, 3.82)$, $(60, 3.75)$, $(50, 3.68)$

$$\frac{3.82-3.75}{70-60} \stackrel{?}{=} \frac{3.75-3.68}{60-50}$$

$$\frac{0.07}{10} \stackrel{?}{=} \frac{0.07}{10} \quad ✓$$