L21: April 13, 2017 (Thursday)

Rocket motion.

Rockets generate thrust through acceleration of a mass of gas through the bottom, propelling the rocket in the opposite direction.

NEWTON'S 3^{RD} LAW: For every action, there is an equal and opposite reaction.

Think: releasing a filled balloon — expelled gas under pressure causes the balloon to zoom around the room...

Def'n: Thrust: a mechanical force caused by the acceleratin of a mass of fluid, in the opposite direc'n of the fluid's flow.

If $v :=$ velocity of rocket

$c :=$ velocity of gas relative to rocket,

Then $c + v :=$ velocity of the gas in space.

Define: $m :=$ mass of rocket (changes with time!)

"up" is the positive direcn

Then THRUST is:

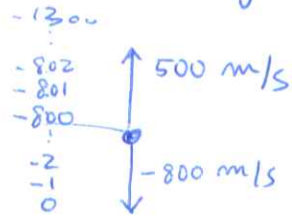
$$\boxed{T = c\,\frac{dm}{dt}}$$

Quick Review 1

(a) ~~Units~~ Dimension of thrust

$$[T] = [c]\left[\frac{dm}{dt}\right] = \frac{\mathcal{L}}{\mathcal{T}}\cdot\frac{\mathcal{M}}{\mathcal{T}} = \frac{\mathcal{L}\cdot\mathcal{M}}{\mathcal{T}^2}$$

(b) Rocket travelling up at 500 m/s, speed of downward gas is 800 m/s. What's c?



$$c = \text{vel. gas in space} - \overbrace{\text{vel. of rocket}}^{v}$$

$$= -800 \text{ m/s} - 500 \text{ m/s}$$

$$= -1300 \text{ m/s}$$

QR1(b) (ct'd)

If "relative to the rocket" means "from the perspective of the rocket",

Then $c = -1300$ (and "velocity of gas in space" means "velocity of gas ~~...~~ from the perspective of an outside observer")

If "rel. to the rocket" means "from the perspective of an outside observer" and "vel. of gas in space" means "from the perspective of the rocket", ~~then~~

Then $c = -800\frac{m}{s} + 500\frac{m}{s} = -300\ m/s.$

QR1(c)

Over 0.1 s, 2 kg of propellant burns. Give the thrust.

$$T = c\frac{dm}{dt} \approx c\frac{\Delta m}{\Delta t} = \left(-300\ \frac{m}{s}\right)\left(\frac{-2\ kg}{0.1\ s}\right)$$

$$= +6000\ \frac{m \cdot kg}{s^2} = +6{,}000\ N.$$

$$\boxed{T = 6{,}000\ N}$$

Impulse: product of thrust with length of time;

Specific impulse: impulse per unit weight of burned fuel,

or: quotient of impulse and change in fuel's weight.

$$I = T\,\Delta t \quad , \quad I := \text{impulse}$$

$$I_{sp} = \frac{I}{\Delta W} = \frac{T\,\Delta t}{\Delta(m \cdot g)} = \frac{T\,\Delta t}{g\,\Delta m} .$$

If we know the specific impulse and solve for T, we obtain:

$$T = I_{sp} \cdot g \cdot \frac{\Delta m}{\Delta t}$$

$$T = \boxed{I_{sp} \cdot g \cdot \frac{dm}{dt}} \qquad (\text{letting } \Delta t \to 0)$$

Thrust is a force that accelerates the rocket, so Newton's 2nd law applies:

$$T = m a$$

$$\boxed{a = \frac{I_{sp}\, g}{m} \cdot \frac{dm}{dt}}$$

← acceleration due to thrust