L4: Jan. 31.

Housekeeping.

- Homework 2/3 due today before class
- Homework 4: Recall "Step 5" of the modelling process — REPORT ON THE MODEL. Do this for the model you submitted for the System Dynamics tutorial. Due in class on Thursday.

Questions?

Today:

- Derivatives
- Differential Eq'ns
- Difference Eq'ns.

MATH 232 · Introduction to Statistics
Autumn 2016

Homework 17: Due in class December 12

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use symbols, you must use them properly. In particular, please avoid the use of the "running equals sign", as this is an abuse of notation and is unacceptable: `http://www.wikiwand.com/en/Equals_sign#/Incorrect_usage`. Write your solutions so that a student one course behind you in the sequence would understand them.

Homework

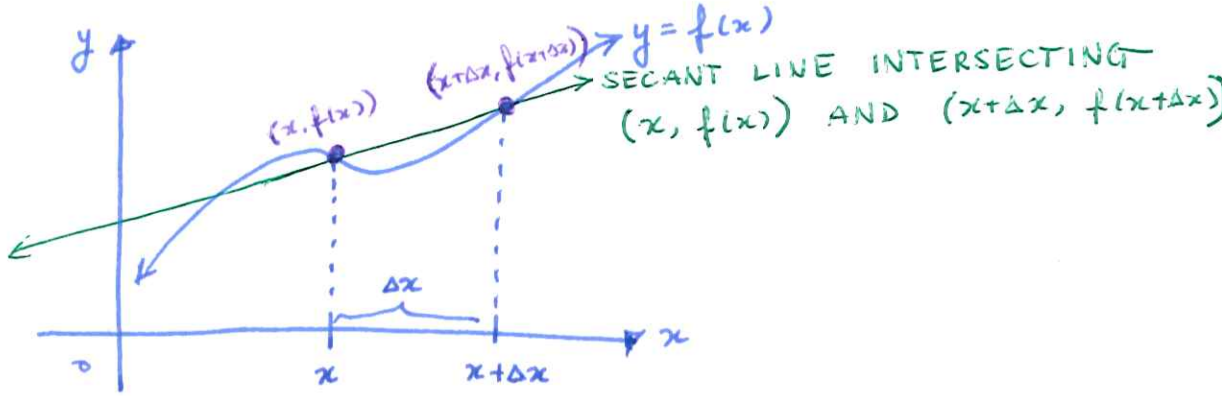
1. For each of the cases below, involving areas under the standard normal curve, decide whether the first area is bigger, the second area is bigger, or the two areas are equal:
   (a) The area to the right of $z = 1.5$ or the area to the right of $z = 2$
   (b) The area to the left of $z = -1.5$ or the area to the left of $z = -2$
   (c) The area to the right of $z = 1$ or the area to the left of $z = -1.5$
   (d) The area to the right of $z = 2$ or the area to the left of $z = -2$
   (e) The area to the right of $z = -2.5$ or the area to the right of $z = -1.5$
   (f) The area to the left of $z = 0$ or the area to the right of $z = -0.1$
   (g) The area to the right of $z = 0$ or the area to the left of $z = 0$
   (h) The area to the right of $z = -1.4$ or the area to the left of $z = -1.4$

2. Find the area under the standard normal curve that lies...
   (a) between $z = 0$ and $z = 0.87$
   (b) Between $z = -1.66$ and $z = 0$
   (c) To the right of $z = 0.48$
   (d) To the right of $z = -0.27$
   (e) To the left of $z = 1.30$
   (f) To the left of $z = -0.79$

[1]See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

· Derivatives & rates of change.

Recall: The secant line that locally intersects two given pts. on a curve:

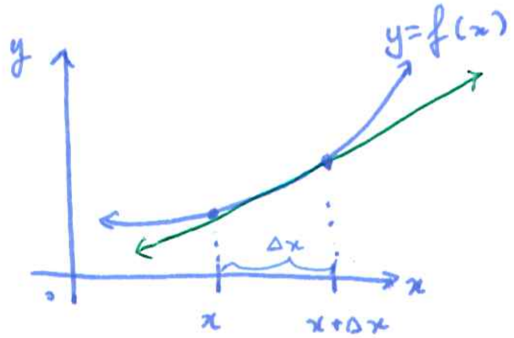


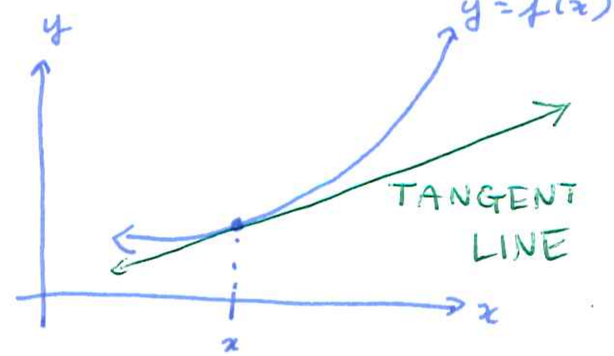
SLOPE OF SECANT LINE ABOVE :

$$m_s = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$$

$$= \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

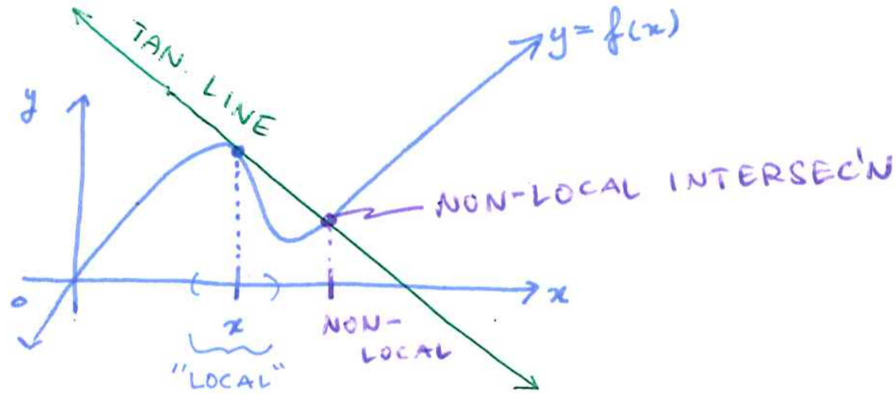
When we let Δx shrink to zero...





A line tangent to ~~[scribbled]~~ the graph of $y=f(x)$ at the point x intersects the graph locally only at the point $(x, f(x))$.

"LOCALLY"?



THE SLOPE OF THE LINE TAN. TO $y=f(x)$ AT x :

$$m(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\left(= \lim_{\Delta x \to 0} m_S(x) \right)!$$

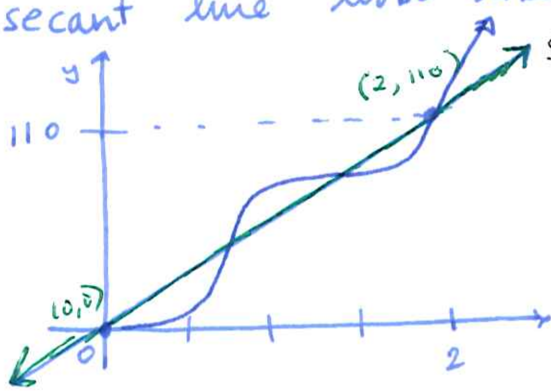
... the slope of the tangent line tells you the <u>instantaneous rate of change</u> of the function values at the point of tangency.

... but sometimes, in math. modelling, the secant line is as good as it gets.

<u>Example</u>. You reset your car or bike's trip odometer to zero (mi / km / lightyears, etc.) at the start of your trip. After 2 hours, you show 110 (mi / km / whatever — units of distance/length).

$$\text{AVERAGE VELOCITY OVER THOSE 2 HRS} = \frac{\Delta(\text{position})}{\Delta(\text{time})} = \frac{\overset{\text{ENDING}}{110\text{ mi}} - \overset{\text{STARTING}}{0\text{ mi}}}{2\text{ hrs} - 0\text{ hrs}} = 55\text{ mph}$$

So, the secant line looks like:



Slope : $m_S = 55$ mph.

The actual pos'n function could have been... anything?

Quick Review 1.

(a) $\text{Avg. velocity} = \dfrac{21.4\,\text{m} - 21.1\,\text{m}}{2\,\text{s} - 1\,\text{s}} = \dfrac{0.3\,\text{m}}{1\,\text{s}} = 30\ \text{cm/sec}$

(b) $\text{Avg. velocity} = \dfrac{11.9\,\text{m} - 21.1\,\text{m}}{3\,\text{s} - 1\,\text{s}} = \dfrac{-9.2\,\text{m}}{2\,\text{s}} = -4.6\,\dfrac{\text{m}}{\text{s}}$

(c) $\text{Avg. velocity} = \dfrac{\Delta s}{\Delta t}$

$$= \frac{s(b) - s(a)}{b - a}$$

$$= \frac{s(b) - s(b - \Delta t)}{\Delta t}$$

(1 sec., 21.1 m)
(3 sec, 11.9 m)

✓ $\Delta s = 11.9\,\text{m} - 21.1\,\text{m} = -9.2\,\text{m}$

✓ $\Delta t = 3\,\text{sec} - 1\,\text{sec} = 2\,\text{sec}$

✓ $b = 3\,\text{sec}$

$a = b - \Delta t = 1\,\text{sec}$

✓ (if $b - \Delta t = 1\,\text{sec}$, then $\Delta t = 2\,\text{sec}$, as $b = 3$.)

✓ $s(b) = 11.9\,\text{m}$

✓ $s(b - \Delta t) = s(a) = 21.1\,\text{m}$

DEF. A differential eq'n is an eq'n that contains derivatives.

Examples. (1) $\frac{dy}{dx} = 1$ is a differential eq'n.

(It is solved by $y = x + c$, any $c \in \mathbb{R}$.)

(2) $\frac{dy}{dx} = y$. (It is solved by $y = Ae^x$, any $A \in \mathbb{R}$.)

Check: $\frac{dy}{dx} = \frac{d}{dx}\left[Ae^x\right] = A\frac{d}{dx}\left[e^x\right] = Ae^x = y$.

(3) $\frac{d^7y}{dx^7} - \left(\frac{d^3y}{dx^3}\right)^3 + \frac{1}{7}y = 5x$.

indep. variable

dep. variable / "unknown" fn.

For the population example... if there is no immigration or emigration, and the populat'n depends only on births & deaths, and those birth/death rates are more or less constant — then the model used to predict population is the MALTHUSIAN MODEL:

$$\frac{dP}{dt} = kP, \quad k \in \mathbb{R} \text{ constant}.$$

- If $k > 0$, then the larger the populat'n, the faster it grows.

- If $k < 0$, then the larger the population, the faster it dies off.

Example. $\underbrace{\frac{dP}{dt} = 0.10P}$, and $\underbrace{P(0) = 100 \text{ bacteria}}_{\text{INITIAL COND'N}}$.

$\frac{dy}{dx} = y$ is solved by $y(x) = Ae^{x}$.

Side Question: what is $\underbrace{\frac{d}{dt}\left[A \cdot e^{kt}\right]}$? $A, k \in \mathbb{R}$.

$= kAe^{kt}$.

So $y(t) = Ae^{kt}$ satisfies $\frac{dy}{dt} = k \cdot y$.

$\underbrace{\frac{dP}{dt} = 0.10P}$.

So $\boxed{P(t) = Ae^{0.10t}}$

"GENERAL" SOL'N.

and $P(0) = 100$ bacteria, by the initial cond'n.

By the general sol'n, $P(0) = A \cdot e^{0.10(0)} = Ae^{0} = A$, so

$A = 100$ satisfies the eq'n.

$$\boxed{P(t) = 100 \cdot e^{0.10t}}.$$