L5; Feb. 2, 2017.

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Housekeeping:

- Model report for System Dynamics tutorial due today!
- Homework for Tuesday to be posted on Canvas
- Tonight 7:30 p.m., Mass MoCA : A Revol'n in 4 Seasons
  $5 students, $5 members, $9 same-day

Last time:

- Def'n of the derivative
- Differential eq'ns
  - Solving "simple" ones, e.g.

$$\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = y$$

$$\frac{dP}{dt} = 0.10 P$$

QUESTIONS?

Today:

- Difference eq'ns
- Computer simulation of quantities governed by difference eq'ns

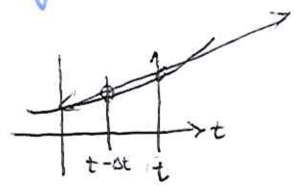
The growth of a quantity is represented by a simple, <u>first order</u> differential equation like

$$\frac{dP}{dt} = (\text{growth}) \; ;$$

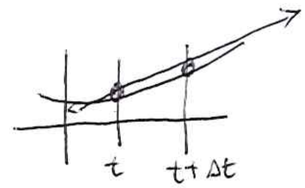
[<u>Recall</u>: in the Malthusian Model, growth $= kP$]

Replacing the derivative with its definition gives:

$$(\text{growth}) = \lim_{\Delta t \to 0} \frac{P(t) - P(t-\Delta t)}{\Delta t}.$$



$$= \lim_{\Delta t \to 0} \frac{P(t+\Delta t) - P(t)}{\Delta t}$$



And, assuming that Δt is small, positive, and fixed, the eq'n can be approximated:

$$\text{growth} \approx \frac{P(t) - P(t-\Delta t)}{\Delta t}$$

or, rearranging:

$$\Delta t \cdot \text{growth} \approx P(t) - P(t-\Delta t), \quad \text{or}$$

$$P(t) \approx P(t-\Delta t) + \Delta t \cdot \text{growth}.$$

This is called a <u>difference equation</u>, or a <u>finite difference equation</u>, and we care about such equations because they give us a way of stepping iteratively through time and tracing the growth (or decay?) of the quantity, EVEN IF WE CANNOT SOLVE THE DIFFERENTIAL EQUATION!

For example...

With the Malthusian model, $\frac{dP}{dt} = 0.1P$, so the growth rate is 0.10 (or 10%), and the growth itself is 0.10 P (or 10% <u>of the current population</u>).

If the population starts out with 100 individuals:

$$P(0) = 100.$$

If we fix a time step $\Delta t := 0.005$ (hours),

then
$$P(0.005) = \underbrace{\overbrace{P(0)}^{100}}_{P(t-\Delta t)} + \underbrace{(0.005)}_{\Delta t}\underbrace{\overbrace{P(0)}^{100}(0.10)}_{\text{growth}}$$

$$= 100 + 0.005 \cdot 100 \cdot 0.10$$