Euler's Identity.

COMPLEX NUMBER is of the form $a+bi$, where $a,b \in \mathbb{R}$ and $i=\sqrt{-1}$.

Notes:

$$\left.\begin{aligned} i^2 &= -1 \\ i^3 &= i^2(i) = -i \\ i^4 &= i^3(i) = (-i)(i) = 1 \\ i^5 &= i^4(i) = i \end{aligned}\right\} \text{cyclic pattern}$$

$$i^6 = i^5(i) = i(i) = -1.$$

Let's substitute $x = i\theta$, $\theta \in \mathbb{R}$, into the Taylor series for e^x:

this should make you nervous – we have not defined exponentiat'n for imag. #'s !!

$$e^{i\theta} = \sum_{k=0}^{\infty} \frac{(i\theta)^k}{k!} = 1 + \frac{i\theta}{1!} + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \cdots$$

$$= 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} - \frac{\theta^6}{6!} - \cdots$$

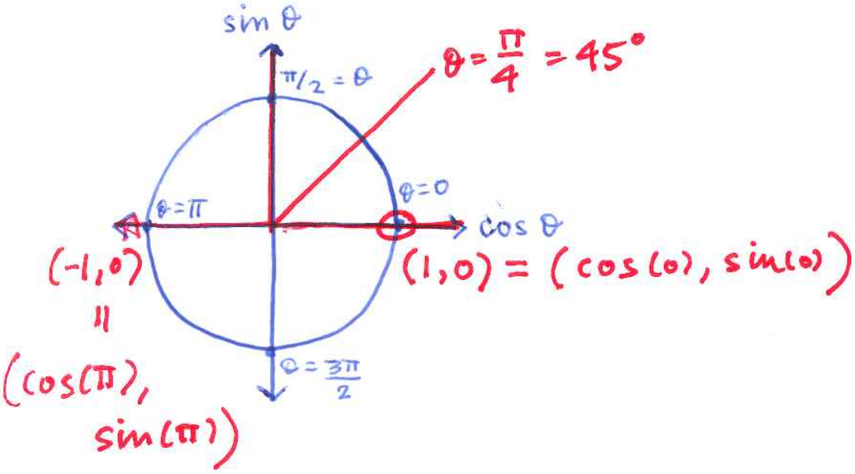
$$= \underbrace{\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots\right)} + i\underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right)}$$

$$= \cos(\theta) + i\sin(\theta).$$

So, $\boxed{\underline{\text{DEFINE}}: \quad e^{i\theta} := \cos(\theta) + i\sin(\theta)}$ ← EULER'S IDENTITY

Euler's identity: $e^{i\theta} := \cos(\theta) + i\sin(\theta)$.

Plug in $\theta = \pi$: $e^{i\pi} = \cos(\pi) + i\sin(\pi)$



$= -1 + i(0)$

$= -1$

Therefore, $e^{i\pi} = -1$ or $\boxed{e^{i\pi} + 1 = 0}$

The 5 most important constants in math! (maybe)